

Dependencies and systemic risk in the European insurance sector: New evidence based on Copula-DCC-GARCH model and selected clustering methods

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ABSTRACT

Objective: The objective of this article is to study the correlations between the most important European insurers and their participation in systemic risk in the insurance sector. We compare systemic risk in different market regimes.

Research Design & Methods: We use statistical clustering methods for time units (weeks) to which we assign conditional variances obtained from the estimated Copula-Dynamic Conditional Correlations-Generalised Auto-Regressive Conditional Heteroskedasticity model (C-DCC-GARCH). In each of the identified market regimes we determine the Conditional Value at Risk *CoVaR* systemic risk measure.

Findings: In this article we show a positive correlation of all the insurance companies under consideration. During global market crises the correlation appears stronger than in 'normal times.' This confirms that the insurance sector generates systemic risk in the presence of turbulences on financial markets, since the value level of the compared index *CoVaR* is much higher in these conditions.

Implications & Recommendations: Our research confirms the insurance sector's contribution to Systemic Risk. Thus, it is important to develop an analysis of systemic risk with a particular attention to the evolution of risk in time and the institutions' interconnectedness in the context of contagion using also some new modelling tools.

Contribution & Value Added: A novel approach of this article is the analysis of dependencies in the insurance sector using the C-DCC-GARCH model with taxonomic methods.

Article type: research article

Keywords: systemic risk; insurance market; Copula-DCC-GARCH(C-DCC-GARCH)

JEL codes: G22, C38, C32

Received: 13 November 2019

Revised: 15 June 2020

Accepted: 20 August 2020

Suggested citation:

Denkowska, A., & Wanat, S. (2020). Dependencies and systemic risk in the European insurance sector. New evidence-based on Copula-DCC-GARCH model and selected clustering methods. *Entrepreneurial Business and Economics Review*, 8(4), 7-27. <https://doi.org/10.15678/EBER.2020.080401>

INTRODUCTION

This article is an answer to the 2017 European Insurance and Occupational Pensions Authority (EIOPA 2017) report that recommends the analysis of systemic risk in the insurance sector; i.e. undesirable financial occurrence with systemic cause and negative global effect in real economy (Eling and Pankoke, 2014 provide 43 definitions of systemic risk). The report pays special attention to two aspects: firstly, the evolution of risk over time and, secondly, dependencies among institutions. In an era of economic globalisation, one of the most important questions is the possibility of financial risk contagion. The higher the level of correlation among insurers, the greater the risk.

Therefore, we aim (i) to analyse systemic risk dynamics for the years 2005-2018, and (ii) to show precisely the interconnectedness among insurers and confirm their impact on systemic risk. The second point comes to the fore through the identified market regime during the largest turbulences on financial markets due to the financial crisis of 2007-2009.

Following the financial crisis of 2007-2009 and the European public debt crisis in 2010-2012, interest in systemic risk has been significantly growing. Among other things, this resulted in the literature proposing many new methods for the study of financial institutions' influence on systemic risk. Moreover, both the academic community and financial regulatory authorities began to pay more attention to the role played by non-bank financial institutions, in particular insurance companies, in creating systemic risk. Before the crisis, most scholars generally accepted that the insurance market has a negligible impact on systemic risk. However – although many a study still supported the latter point of view – the recent literature offers several articles suggesting the possibility of the insurance market itself creating systemic risk. Let us quote here from a few articles whose authors claim that insurance companies:

- generate systemic risk (Billio, Getmansky, Lo, & Pelizzon, 2012; Weiß & Mühlnickel, 2014),
- can be systemically important when they conduct investment activities outside of their normal insurance business (Baluch, Mutenga, & Parsons, 2011; Cummins & Weiss, 2014), while in general the systemic significance of the insurance sector as a whole is still subordinated to the banking sector (Chen *et al.*, 2013; Czerwińska, 2014),
- are systemically unimportant due to the low level of interconnections and the lack of strong dependence on external funding (Harrington, 2009; Bell, 2009; Keller, 2009; Geneva Association, 2010).

On the other hand, after studying a very large sample of insurers in a long-term horizon, Bierth, Irresberger and Weis (2015) claim that the level of generating systemic risk by the insurance sector is rather low, its peak having been reached during the financial crisis of 2007-2009. Moreover, these authors indicate the four L's – linkages, leverage, losses, liquidity – as the crucial factors influencing the exposure of insurers to systemic risk.

The present article belongs to the mainstream of studies in the linkages among large insurance companies and their participation in systemic risk in the insurance sector. Our main aim is to check whether the strength of existing connections among the eight largest insurers depend on the insurance market regime. These eight companies come from the list of the most important insurance companies in the world with respect to total assets – five from Europe, one from the USA, Canada, and China – together with their participation

in systemic risk in the European insurance sector. The market regimes are identified by analysing the weekly rates of return of the insurers in question during the period between January 2005 and December 2018. They are assessed using statistical clustering methods of time units (weeks) to which we assigned conditional variances obtained from the estimated C-DCC-GARCH model. Indeed, we assume that the change (increase) of the risk (variance) is a good and classical index of the financial market tension. Such an approach has the advantage that there is no need to assume a priori a number of market regimes, because this number is identified by the clustering quality assessment. Next, in each of the identified regimes we establish the CoVaR systemic risk measure, commonly used today (see e.g. Acharya, Pedersen, Philippon, & Richardson 2010; Bierth *et al.*, 2015; Jobst, 2014). We assume that the European insurance market is represented by the weekly rates of return from the STOXX 600 Europe Insurance index. The CoVaR measure, indicating the participation of each of the insurers to systemic risk, is assessed using the conditional distributions obtained from eight bivariate C-DCC-GARCH models. In each of these models one boundary distribution represents the European insurance market – on the logarithmic return from the stock market index STOXX 600 Europe Insurance index – while the other one represents the insurer, on the appropriate logarithmic rate of return. To the best of our knowledge, such an approach has not been used in systemic risk analysis ever before.

The paper consists of five chapters. The second one overviews the literature devoted to systemic risk in the insurance sector, the third chapter presents the methodology together with empirical results, the fourth one shows the data and describes our findings, whereas the fifth and last one proposes conclusions.

LITERATURE REVIEW

Let us begin with recalling the natural definition of systemic risk as ‘any set of circumstances that threatens the stability of or public confidence in the financial system’ (Billio *et al.*, 2012).

Usually, systemic risk is endogenous, i.e. coming from the financial system itself, which amplifies its exogenous version. Systemic risk can be viewed as a coordination failure. The specific sources of systemic crisis are contagion, bank run, or liquidity crisis. Up to now, insurance has virtually been immune to systemic risk, which is partly explained by pyramidal risk sharing – which removes a lot of contagion risk – and less room for coordination failure than in other financial institutions. However, as insurance companies become increasingly involved in other financial activities or – rather – as insurance is increasingly often conducted by financial institutions that do not specialise only in this sector, the situation may well change. Of course, there are other causes that may lead to this, such as e.g. more pervasive liquidity insurance offer by the companies. In particular, these conclusions can be found in the special report by the Geneva Association (2010), ‘Systemic risk in insurance: An analysis of insurance and financial stability’. Furthermore, Billio *et al.* (2010) already mention the growing interrelations between the insurance, banking, and hedge funds sectors as one of the causes of increasing systemic risk.

Another question is how to measure systemic risk, as several approaches are possible. Leaving this question aside for the moment – as the matter is raised in many of the articles mentioned below (e.g. Bernardi & Catania, 2015) – let us quick overview at recent approaches to systemic risk in insurance. The general and most widespread view is that, for various reasons, the added value of insurance sector to systemic risk – whatever

its definition and measurement tools – is very low but this recently undergoes a change, as the insurance market keeps evolving (also cf. the 2015 ‘Report on systemic risks in the EU insurance sector’ by ESRB, 2015).

Indeed, Kanno (2016) observes that – contrary to the interbank market – the insurance market does not contain feedback mechanisms that would make it fully interconnected. However, Kanno indicates that interconnectedness in the insurance sector has not been explored yet with network theory or contagious default approach. As a conclusion, Kanno upholds the opinion of International Association of Insurance Supervisors (IAIS, 2011) that the degree of interconnectedness within the (re)insurance sector is small, which adds to its immunity to systemic risk. However, an earlier study (Dungey, Luciani, & Veredas, 2014) notes that insurance companies display substantial systemic risk via interconnectedness with the financial sector and the real economy. Similarly, Bierth, Irresberger, and Weiß (2015) studied the contribution of 253 international life and non-life insurers to global systemic risk in 2000-2012, and they observe that systemic risk in the international insurance sector is small in comparison to that of banks. Still, during the financial crisis, insurers significantly contributed to the instability of the financial sector. In conclusion, the various factors determining the systemic risk of insurers are interconnectedness, leverage, loss ratios, and the insurer’s funding fragility. Bierth, Irresberger, and Weiß (2015) furthermore conclude that there is no big difference in the contribution to global systemic risk between life insurers and non-life insurers. In particular, there seems to be no relationships between an insurer’s size and its contribution. The authors support the viewpoint that unlike the banking sector, the insurance one predominantly suffers from exposition to systemic risk, rather than from the financial system’s fragility. Moreover, another study (Mühlnickel & Weiß, 2015) indicates a strong positive relationship between consolidation in the insurance industry and moderate systemic risk in the insurance sector, but definitely no extreme systemic risk. Similar conclusions are drawn by Berdin and Sotocornola (2015), who use three measurements to infer that the insurance industry has a persistent systemic relevance over time but far from the role of banks in causing systemic risk compared to banks. An interesting contrast between the Eurozone and the USA is observed by Bernal, Gnabo, and Guilmin (2014), who surmise that in 2004-2012, the other financial services sector and the banking sector in the Eurozone contribute relatively more to systemic risk in periods of distress than the insurance sector, while in the USA the insurance industry is systemically the riskiest financial sector.

These recent results were preceded by several articles – many of them triggered by the AIG’s collapse in the recent crisis – in the years 2009-2013 (as listed in the excellent survey by Eling and Pankoke, 2014). Harrington (2009) claims that traditional insurance products make no contribution to systemic risk. Radice (2010) comes to a two-fold conclusion. He identified those phenomena that do not contribute to generating SR; According to him, these are the unavailability of insurance, life insurance, insolvency of CDS and the use of credit ratings. He indicated those that may be systemically risky, i.e. contagion with assets, limited fungibility of the available liquidity of the group, difficulties in unregulated / uninsured activity within the insurance group.

Baluch, Mutenga, and Parsons (2011) noted that the increase in systemic risk in the insurance sector has been caused in recent years by an increased share in capital markets and the introduction of banking services.

The same year, van Lelyveld, Liedorp, Kampman van Lelyveld, Liedorp and Kampman (2011) studied contagion and the contribution of linkages among insurers and reinsurers to systemic failure, which leads them to conclude that the collapse of several reinsurers would result in the bankruptcy of only a few primary insurers.

That is, these authors suggest that the potential failure of one or more (re)insurers is not a systemic risk. Still, in 2011, a study of the US insurance sector was performed by Cummins and Weiss (2014a), which shows that the largest contributors to SR are non-traditional and non-insurance activities such as derivatives trading and financial guarantees.

Grace (2011) states that the situation in the insurance sector is different from that in the banking sector; the duration of assets and liabilities are more closely matched.

Similarly, Kessler (2013) asserts that reinsurance does not contribute to systemic risk, while Baur, Enz, and Zanetti (2003) come to the same conclusion. On the other hand, Mühlnickel and Weiß (2014) claim that the insurance sector is sensitive to the financial system's deterioration and contributes to systemic risk.

Schwarcz and Schwarcz (2014) concentrate on systemic risk in insurance as resulting from correlations among firms.

Our work responds to the problems still open in literature (Brechmann *et al.*, 2013; Reboredo & Ugolini, 2015; Di Bernardino *et al.*, 2015) regarding the analysis of the insurance sector in the context of interrelationships and systemic risk along with SR measures (Barrieu *et al.*, 2014; Tang & Yang, 2012). We undertake research both in the context of searching for a model and assessing whether and at what level SR is generated during the normal state of the market and during turbulences. We analyse eight insurance companies from the list of the most important insurance companies in the world ranked by total assets, five of which are the largest in Europe, two in North America, and one in Asia. Thus, we propose the following research hypotheses:

1. All analysed insurers generate systemic risk in the European insurance sector regardless of the country, currency, and the size of insurer measured by the size of assets.
2. The systemic importance of the European insurance market is the same for all insurers, except for the Chinese, for whom it is less important (CoVaR is higher). During turbulences, the SR generation level is much higher than in the normal state.
3. The existence of strong relationships between insurers and the European insurance sector results in a higher SR level.

In order to verify the hypotheses, an innovative hybrid approach has been used, which combines machine learning cluster analysis with the C-CDD-GARCH model. We used the C-DCC-GARCH model (Di Clemente, 2018; Karimalis & Nomikos, 2018; Oh & Patton, 2018; Gaizner, 2019) in three different contexts:

- in combination with cluster analysis methods to determine market states which – as far as we know – has not been described in literature up to now,
- to determine conditional correlations between insurers,
- to calculate the CoVaR risk measure.

RESEARCH METHODOLOGY

The empirical strategy we use in this article to analyse the dependences and assess systemic risk on the European insurance market consists of two basic steps:

1. Market regime identification;
2. Analysis of identified market regimes:
 - dependences among the studied insurance companies,
 - correlations between a given insurance company and the European insurance market as represented by the STOXX 600 Europe Insurance index,
 - systemic risk.

It is assumed in the first step that market regimes are identified using statistical methods of grouping weekly periods t according to the assigned conditional variances of rates of return of all the instruments under analysis. The conditional variances that are essential in this approach are obtained through the multivariate C-DDC-GARCH model. In this model, the distribution of the rates of return vector $r_t = (r_{1,t}, \dots, r_{k,t})'$ – conditional with respect to the set Ω_{t-1} of information available up to the moment $t - 1$ – is modelled using the conditional copula proposed by Patton (2006). The copula assumes the following form:

$$r_{1,t} | \Omega_{t-1} \sim F_{1,t}(\cdot | \Omega_{t-1}), \dots, r_{k,t} | \Omega_{t-1} \sim F_{k,t}(\cdot | \Omega_{t-1}) \quad (1)$$

$$r_t | \Omega_{t-1} \sim F_t(\cdot | \Omega_{t-1}) \quad (2)$$

$$F_t(r_t | \Omega_{t-1}) = C_t(F_{1,t}(r_{1,t} | \Omega_{t-1}), \dots, F_{k,t}(r_{k,t} | \Omega_{t-1})) \quad (3)$$

in which C_t denotes the copula, whereas F_t and $F_{i,t}$ are the multivariate CDF and the CDFs of the marginal distributions at time t . In general, the univariate rates of return $r_{i,t}$ can be modelled by various specifications of the mean model, e.g. the ARMA process (Box & Jenkins, 1970) and various specifications of the variance model e.g. sGARCH, fGARCH, eGARCH, gjrGARCH, apARCH, iGARCH, csGARCH (Fiszeder, 2009).

In our study, the following ARMA process is applied to all the series of returns for the mean:

$$r_{i,t} = \mu_{i,t} + y_{i,t}, \quad (4)$$

$$\mu_{i,t} = E(r_{i,t} | \Omega_{t-1}), \quad \mu_{i,t} = \mu_{i,0} + \sum_{j=1}^{p_i} \varphi_{ij} r_{i,t-j} + \sum_{j=1}^{q_i} \theta_{ij} y_{i,t-j}, \quad (5)$$

$$y_{i,t} = \sqrt{h_{i,t}} \varepsilon_{i,t}, \quad (6)$$

While for the variance we use the eGARCH model (Nelson, 1991):

$$\log(h_{i,t}) = \omega_i + \sum_{j=1}^{p_i} (\alpha_{ij} \varepsilon_{i,t-j} + \gamma_{ij} (|\varepsilon_{i,t-j}| - E|\varepsilon_{i,t-j}|)) + \sum_{j=1}^{q_i} \beta_{ij} \log(h_{i,t}), \quad (7)$$

In which $\varepsilon_{i,t} = y_{i,t} / \sqrt{h_{i,t}}$, are independent random variables with the same distribution. In the empirical analysis we considered the following distributions: normal, skew-normal, t-Student, skew-t-Student and GED.

The structure of the dependences between the rates of return is modelled using elliptic copulae with conditional correlations R_t as parameters, the dynamics of which is described by the DCC(m, n) model:

$$H_t = D_t R_t D_t \quad (8)$$

$$D_t = \text{diag}(\sqrt{h_{1,t}}, \dots, \sqrt{h_{k,t}}), \quad (9)$$

$$R_t = (\text{diag}(Q_t))^{-\frac{1}{2}} Q_t (\text{diag}(Q_t))^{-\frac{1}{2}}, \quad (10)$$

$$Q_t = (1 - \sum_{j=1}^m c_j - \sum_{j=1}^n d_j) \bar{Q} + \sum_{j=1}^m c_j (\varepsilon_{t-j} \varepsilon'_{t-j}) + \sum_{j=1}^n d_j Q_{t-j}, \quad (11)$$

in which the conditional variances $h_{i,t}$ are modelled using univariate GARCH(p,q) processes of the form (7), $\varepsilon_t = D_t^{-1}y_t$, $y_t = (y_{1,t}, \dots, y_{k,t})'$ and \bar{Q} is the unconditional covariance matrix of standardised residuals ε_t . In the specification (11) c_j ($j = 1, \dots, m$), d_j ($j = 1, \dots, n$) are scalars describing the influence on the current correlations of the respective previous shocks and previous conditional correlations.

The parameters of the C-DCC-GARCH model above are estimated using the *inference function for margins – IFM*. This method is presented in detail e.g. in Joe (1997). The computations were done in the R environment using the 'rmgarch' package developed by Ghalanos (2019).

We used statistical methods of unsupervised classification in order to identify market regimes. We assumed that the groups obtained from periods t have similar levels of risk, i.e. have a similar conditional variance. The clustering was performed by means of hierarchical methods in which groups are created recursively by connecting the most similar objects (Ward's method). We also used two division methods, i.e. the classical k-means method and the partitioning around medoids method (PAM) proposed by Kaufman and Rousseeuw (1990). The optimal number of groups – and thus the market regimes – were assessed under the following measures of cluster validity: the *Calinski-Harabasz index* (Calinski & Harabasz, 1974), the *silhouette index-SI* (Kaufman & Rousseeuw, 1990), the *Dunn index* (Dunn, 1974), and the *Xie-Beni separation measure* (Xie & Beni, 1991).

In the second stage of analysis, in each of the identified market regimes we assessed the *CoVaR*. The systemic risk measure $CoVaR_{\beta,t}^{ij}$ was defined to be the value at risk (*VaR*) of the institution (market index) i under the condition that another institution (market index) j is subject to distress, i.e. its rate of return is smaller than its value at risk, meaning that:

$$P\left(r_{i,t} \leq CoVaR_{\beta,t}^{ij} | r_{j,t} \leq VaR_{\alpha,t}^j\right) = \beta, \quad (12)$$

Using the conditional probability formula we received:

$$\frac{P\left(r_{i,t} \leq CoVaR_{\beta,t}^{ij}, r_{j,t} \leq VaR_{\alpha,t}^j\right)}{P\left(r_{j,t} \leq VaR_{\alpha,t}^j\right)} = \beta, \quad (13)$$

The definition of the value at risk for the institution j , i.e. $VaR_{\alpha,t}^j$ yielded $P\left(r_{j,t} \leq VaR_{\alpha,t}^j\right) = \alpha$, that is:

$$P\left(r_{i,t} \leq CoVaR_{\beta,t}^{ij}, r_{j,t} \leq VaR_{\alpha,t}^j\right) = \alpha\beta. \quad (14)$$

Therefore, the assessment of $CoVaR_{\beta,t}^{ij}$ required the knowledge of bivariate distribution F_t of the vector $(r_{i,t}, r_{j,t})$. Due to the Sklar Theorem, this distribution can be represented using the copula in the following way:

$$F_t(r_{i,t}, r_{j,t}) = C_t\left(F_i(r_{i,t}), F_j(r_{j,t})\right). \quad (15)$$

Invoking (15), $CoVaR_{\beta,t}^{ij}$ can be determined by (numerically) solving the equation:

$$C_t\left(F_i\left(CoVaR_{\beta,t}^{ij}\right), \alpha\right) = \alpha\beta. \quad (16)$$

In the empirical analysis, we studied the influence on the European insurance market's systemic risk of the five largest insurance companies from Europe and the biggest

insurers from the USA, Canada, and China. We assumed that $r_{i,t}$ represents the European insurance market (we made use of the weekly rates of return from STOXX 600 Europe Insurance), while $r_{j,t}$ describes the insurers (we made use of the weekly logarithmic returns on shares). For each of the eight pairs – the rate of return from the STOXX 600 index $r_{i,t}$, logarithmic return of the insurer $r_{j,t}$ – we assessed parameters of the bivariate C-DCC-GARCH model described by the formulae (1)-(7). Then, using these parameters together with the conditional correlations obtained by these models, we determined the copula C_t and the distributions F_t . The values $CoVaR_{\beta,t}^{ij}$ for the analysed period were obtained by solving numerically the equation (16).

RESULTS AND DISCUSSION

From the literature analysis, we conclude that much was already written about the generation of SR in the insurance sector, and the conclusions are divided. Baluch, Mutenga, and Parsons (2011) and Schwarcz and Schwarcz (2014) confirm the thesis that the insurance sector generates SR, especially in the recent period, when insurers have expanded non-insurance activities. From our case study of the eight largest insurers, we conclude that each insurance company generates SR. In addition, the SR level increases during turbulences on financial markets. Many works (e.g. Barrieu *et al.*, 2014; Tang & Yang, 2012; Jobst, 2014) state that tools for measuring SR have not been developed yet, the universal definition of the SR measure has not yet been established. Oh and Patton (2018) and Reboredo and Ugolini (2015) show that the C-DCC-GARCH model enables the study of SR in the banking sector, considering turbulences on financial markets. In our work, we confirm that the model also works in the insurance sector. The novelty of our article lies in the creation of a model that combines taxonomic methods with the econometric C-CDD-GARCH model. As the basis for our study, we took stock prices of the five largest insurers from Europe and the biggest insurers from the USA, Canada, and China (cf. Table 1 and Figure 1), along with the STOXX 600 Europe Insurance index representing the European insurance market (cf. Figure 2). Data were obtained from the Thomson Reuters in January 2019. We analysed the weekly log returns for the period between January 2005 and December 2018.

Table 1. Insurance companies considered in the study with their acronyms used in the presentation of results

No.	Insurer	Acronym	Country	Total assets (in bln USD)
1	AXA	AXA	France	944.145
2	Allianz	Allianz	Germany	934.654
3	Prudential plc	Prud	United Kingdom	578.149
4	Legal & General	Legal	United Kingdom	574.901
5	Aviva	Aviva	United Kingdom	541.188
6	Metlife	Metlife	USA	898.764
7	Manulife Financial	Manu	Canada	534.705
8	Ping An Insurance	Ping	China	802.975

Source: own elaboration based on of data from <http://www.relbanks.com/top-insurance-companies/world> (15 January 2019).

All insurers besides Legal & General and Manulife Financial are listed by G-SII according to the principles suggested by the Association of Insurance Supervisors (IAIS) in Basel in 2013, which established how to evaluate financial institutions as far as systemic importance is concerned.

In the first stage of our study, we identified the regimes of insurance market on the basis of the conditional variances of rates of return of the insurance companies under scrutiny. We assessed these conditional variances using the eight-variate C-DCC-GARCH model. During the analysis, we considered various ARMA-GARCH specifications of univariate models. Finally, on the grounds of information criteria and model appropriateness tests (result available upon request to the authors), we opted for all the instruments, i.e. for the ARMA(1, 1)-eGARCH(2, 2) model with the skew Student distribution (with skewness ξ and shape ν); the eGARCH meaning exponential GARCH model put forward by Nelson. During the analysis of the dynamics of dependences between the rates of return, we considered the Gauss and Student copula together with various specifications of the DCC model. As earlier, on the basis of information criteria, we chose the Student copula with conditional correlation coefficients obtained from the DCC(1, 1) model and a constant shape parameter η . The assessment results are presented in Table 2, while the conditional variances obtained are shown in Figure 3.

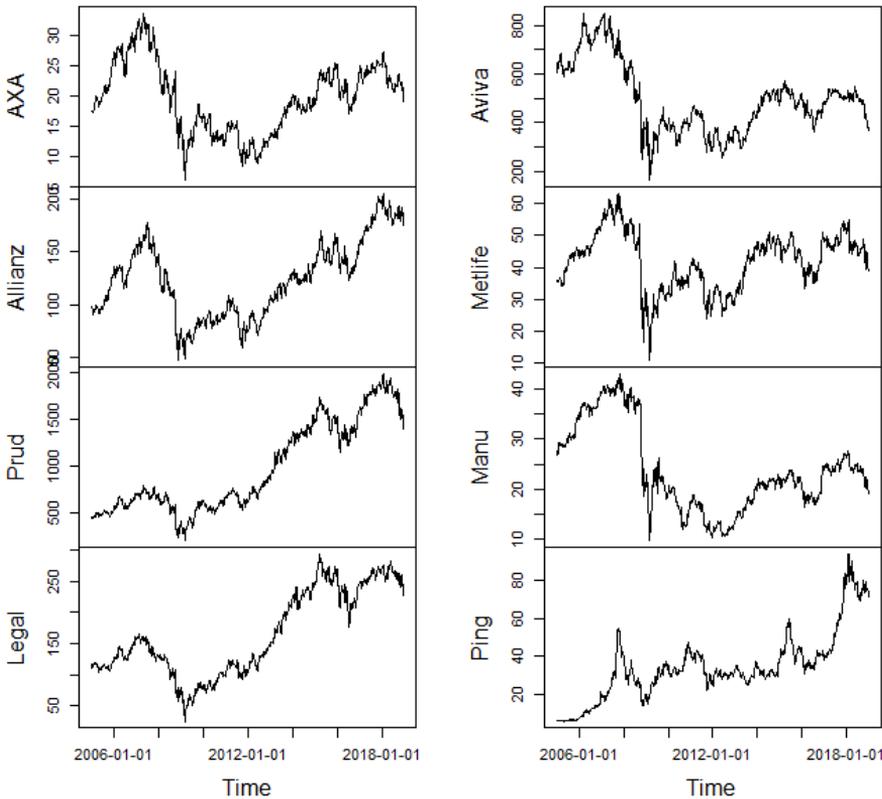


Figure 1. Quotations of insurance companies studied for the period 07.01.2005-21.12.2018
 Source: own elaboration.

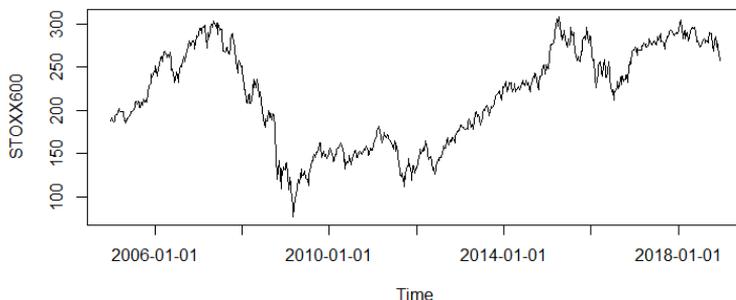


Figure 2. STOX 600 Europe Insurance index during the period 07.01.2005-21.12.2018

Source: own elaboration.

Table 2. C-DCC–GARCH model estimation results

Param.	AXA	Allianz	Prud	Legal	Aviva	Metlife	Manu	Ping
M	0.0010 <i>0.3569</i>	0.0011 <i>0.3294</i>	0.0009 <i>0.2815</i>	0.0011 <i>0.0604</i>	-0.0006 <i>0.5609</i>	0.0009 <i>0.0930</i>	0.0003 <i>0.8047</i>	0.0039 <i>0.0198</i>
φ_1	0.8445 <i>0.0000</i>	0.2844 <i>0.0000</i>	0.6072 <i>0.0000</i>	0.7425 <i>0.0000</i>	0.7211 <i>0.0000</i>	0.7876 <i>0.0000</i>	-0.8687 <i>0.0005</i>	-0.9367 <i>0.0000</i>
ϑ_1	-0.8897 <i>0.0000</i>	-0.3397 <i>0.0000</i>	-0.7336 <i>0.0000</i>	-0.8123 <i>0.0000</i>	-0.7797 <i>0.0000</i>	-0.8440 <i>0.0000</i>	0.8056 <i>0.0073</i>	0.9118 <i>0.0000</i>
Ω	-0.1891 <i>0.0000</i>	-0.2024 <i>0.0001</i>	-0.1287 <i>0.0011</i>	-0.1854 <i>0.0282</i>	-0.2492 <i>0.0040</i>	-0.1608 <i>0.0041</i>	-0.2018 <i>0.0123</i>	-0.2718 <i>0.0848</i>
α_1	-0.3000 <i>0.0000</i>	-0.2597 <i>0.0000</i>	-0.1974 <i>0.0003</i>	-0.2058 <i>0.0191</i>	-0.1721 <i>0.0003</i>	-0.1963 <i>0.0000</i>	-0.1868 <i>0.0097</i>	-0.0396 <i>0.3617</i>
α_2	0.2021 <i>0.0002</i>	0.1564 <i>0.0125</i>	0.0907 <i>0.1053</i>	0.1036 <i>0.3209</i>	-0.0184 <i>0.7349</i>	0.0810 <i>0.0084</i>	0.0225 <i>0.7240</i>	0.0356 <i>0.4804</i>
β_1	1.0000 <i>0.0000</i>	1.0000 <i>0.0000</i>	1.0000 <i>0.0000</i>	1.0000 <i>0.0000</i>	0.1931 <i>0.0000</i>	1.0000 <i>0.0000</i>	0.5951 <i>0.0000</i>	0.2430 <i>0.0000</i>
β_2	-0.0299 <i>0.0000</i>	-0.0300 <i>0.0001</i>	-0.0205 <i>0.0009</i>	-0.0284 <i>0.0376</i>	0.7682 <i>0.0000</i>	-0.0252 <i>0.0044</i>	0.3751 <i>0.0000</i>	0.7113 <i>0.0000</i>
γ_1	-0.0366 <i>0.6340</i>	0.0532 <i>0.6021</i>	-0.0929 <i>0.2804</i>	0.1295 <i>0.2064</i>	0.0277 <i>0.7292</i>	0.1169 <i>0.2584</i>	0.1414 <i>0.1533</i>	0.2838 <i>0.0000</i>
γ_2	0.1790 <i>0.0334</i>	0.0507 <i>0.6303</i>	0.2159 <i>0.0186</i>	0.0764 <i>0.4639</i>	0.3062 <i>0.0001</i>	0.0103 <i>0.9199</i>	0.0824 <i>0.3380</i>	0.0932 <i>0.2080</i>
ξ (skew.)	0.8519 <i>0.0000</i>	0.8332 <i>0.0000</i>	0.8022 <i>0.0000</i>	0.8906 <i>0.0000</i>	0.8152 <i>0.0000</i>	0.8709 <i>0.0000</i>	0.9219 <i>0.0000</i>	1.1321 <i>0.0000</i>
ν (shape)	11.7322 <i>0.0118</i>	10.1324 <i>0.0079</i>	6.0408 <i>0.0000</i>	5.4374 <i>0.0000</i>	6.0600 <i>0.0000</i>	4.5168 <i>0.0000</i>	5.0649 <i>0.0000</i>	5.4819 <i>0.0000</i>
C-DCC parameters								
Distribution	Octovariate t-Student							
DCC order	DCC(1, 1)							
	Parameters							
c_1	0.01063 (<i>0.00012</i>)							
d_1	0.94801 (<i>0.00000</i>)							
η (shape)	9.96436 (<i>0.00000</i>)							

The numbers in parentheses are probability values (p-values).

Source: own study.

Market regimes were identified by means of clustering weekly periods with respect to the conditional variances in insurance companies' rates of return. In this crucial step – from the viewpoint of the whole study – we considered various combinations of distance measures, clustering methods, and a number of classes. Eventually, following criteria of clustering quality (cf. Table 3), we chose a division into two classes obtained using the method of k-means with the Euclidean distance (cf. Figure 4). In this case, the silhouette index is 0.8683 (clustering quality is pictured in Figure 5). We assumed that different market regimes correspond to different classes. The variance distribution in different regimes is shown in Figure 6. We can infer from Figure 6 that the first regime is characterised by low volatility (low risk level), while the second one – occurring during the period 17.10.2008-05.06.2009 – by high volatility (high risk level).

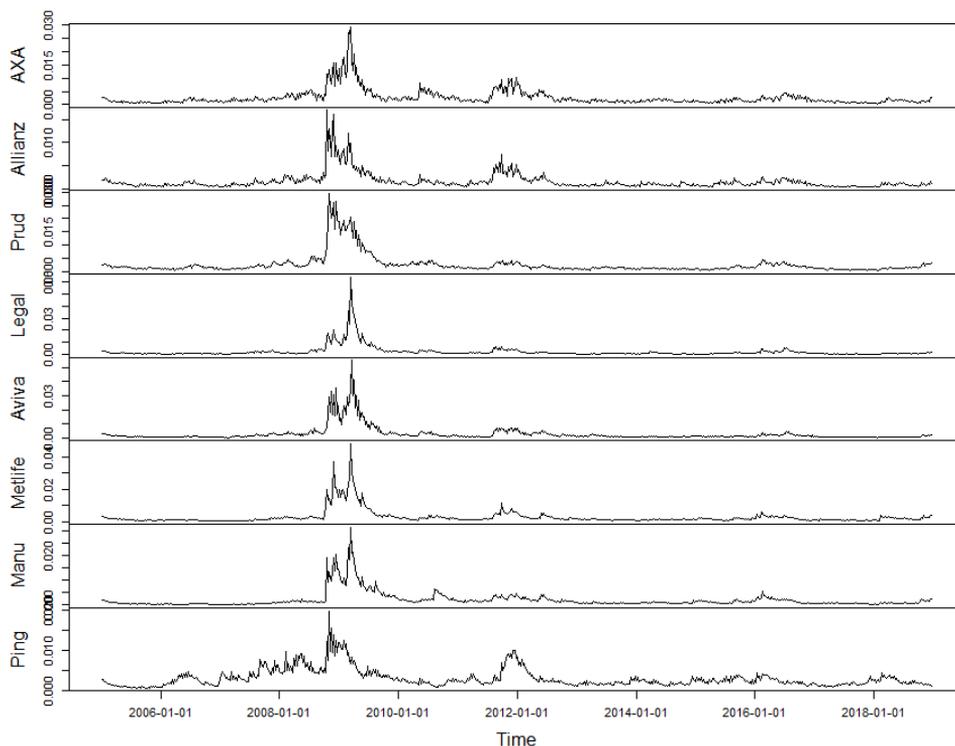


Figure 3. Conditional variances

Source: own elaboration.

In the second step of our study, we analysed dependences between the studied insurance companies based on the conditional correlations from the previously assessed octovariate c-DCC-GARCH model. Their distribution for the respective pairs in the identified market regimes is shown in Figure 7.

On the other hand, the analysis of dependences between the insurer and the European insurance market – but also the analysis of systemic risk in the first and second market regime – was conducted on the basis of the estimated eight-bivariate C-DCC-GARCH

models for the following pairs: the rate of return on the European market index and the individual rate of return for a given insurance company. The models were evaluated on the basis of the whole history of occurrences. In the case of insurers, we employed the earlier estimated ARMA(1, 1)-eGARCH(2, 2) models with the skew Student distribution. On the grounds of information criteria and model appropriateness tests, we considered the same specification for the STOXX 600 Europe Insurance index rate of return. The parameters of the estimated model are given in Table 4. During the analysis of the

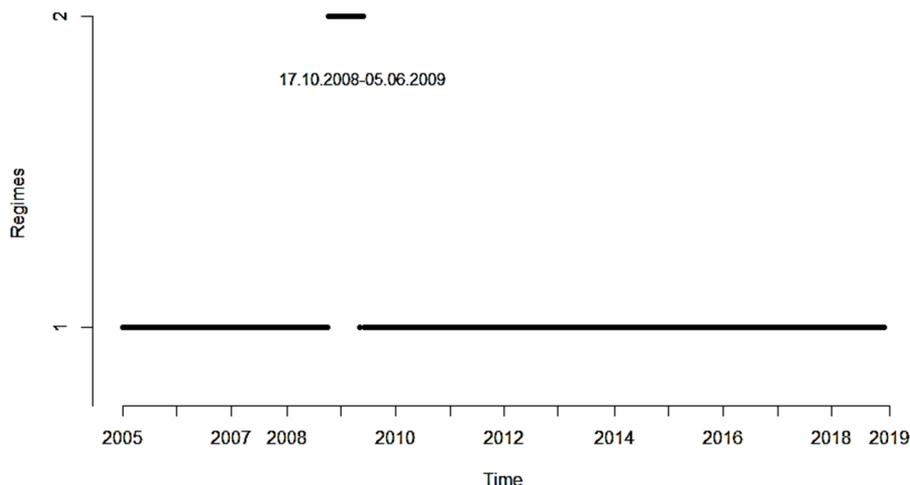


Figure 4. Identified market regimes

Source: own elaboration.

Table 3. Validation indices for data partitions

Validation criterion	Number of clusters				
	2	3	4	5	6
	Ward's method				
Silhouette	0.8683	0.4202	0.3958	0.3987	0.3986
Calinski Harabasz index	1545.1570	1006.8530	771.5901	963.3596	814.7552
Dunn index	0.0552	0.0080	0.0080	0.0110	0.0110
Xie-Beni index	1.9208	76.1650	68.5520	45.4610	43.3223
	PAM				
Silhouette	0.8623	0.4788	0.4153	0.4181	0.1549
Calinski Harabasz index	1501.2950	1036.3830	791.2769	990.6590	809.8822
Dunn index	0.0353	0.0082	0.0077	0.0104	0.0053
Xie-Beni index	4.1444	66.2503	72.2987	47.7384	177.4645
	k-means				
Silhouette	0.8683	0.5238	0.5177	0.4713	0.4394
Calinski Harabasz index	1545.1570	1063.6570	1170.1440	1047.2740	915.3568
Dunn index	0.0552	0.0071	0.0106	0.0146	0.0127
Xie-Beni index	1.9208	92.8171	62.4426	28.7042	34.8416

Note: numbers in bold indicate the optimal number of groups with reference to a given criterion.

Source: own study.

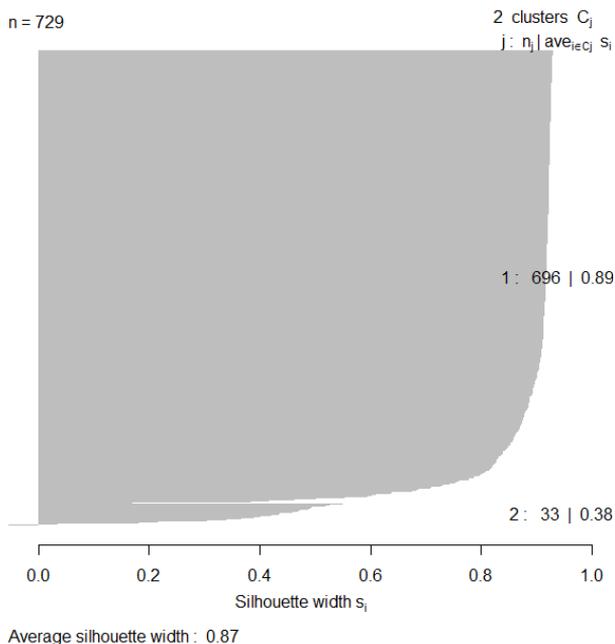


Figure 5. Silhouette plot
 Source: own elaboration.

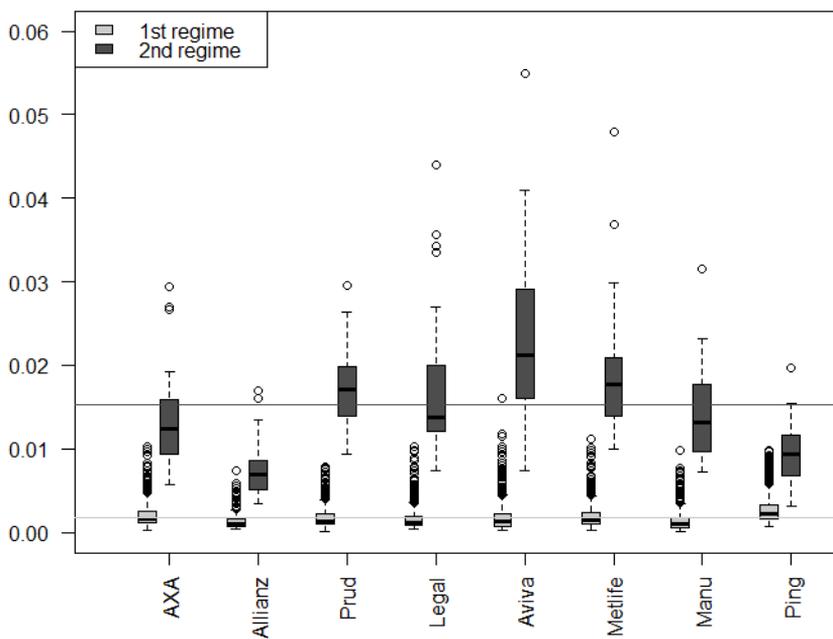


Figure 6. Boxplots for the conditional variance in the identified market regimes
 Source: own elaboration.

dynamics between the rate of return on the index representing the European insurance market and the insurers' rates of return, we considered the Gauss and Student copulae, along with various specifications of the DCC model. On the basis of information criteria for each pair, we chose the Student copula with conditional correlations obtained from the DCC(1, 1) model and constant shape parameters. The estimation results are presented in Table 5, while the conditional correlations obtained are shown in Figure 8. Finally, the distribution of the conditional correlations between the domestic and European capital markets in the identified regimes is given in Figure 9.

Systemic risk assessment in identified market regimes was performed using the *CoVaR* measure determined by the method described in the previous section. The *CoVaR* value distribution illustrating the influence of a given insurer on the European insurance market is shown in Figure 10.

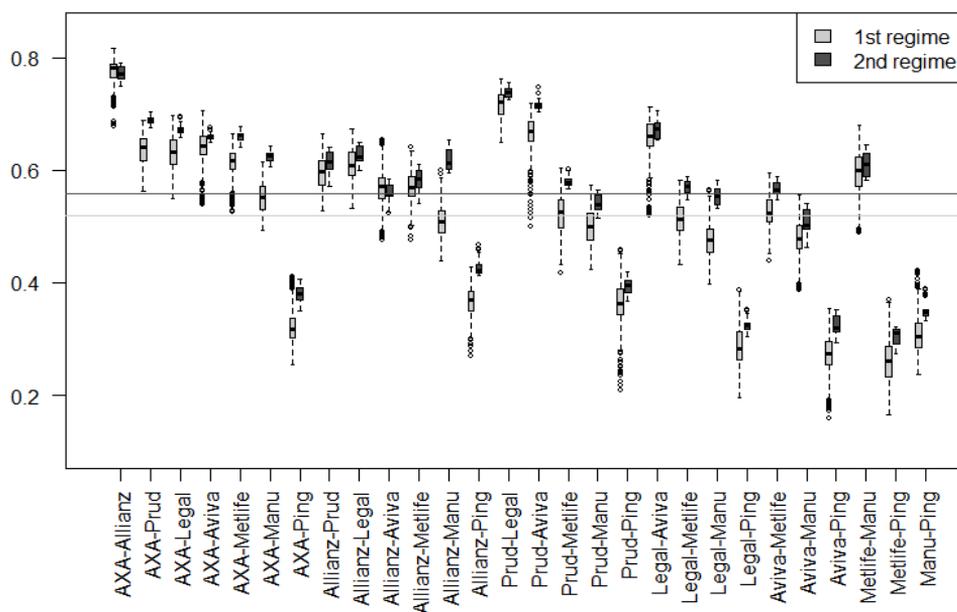


Figure 7. Boxplots for the conditional correlations between analysed markets in the identified regimes

Source: own elaboration.

Market regimes were established to check whether the systemic importance of the surveyed largest insurers from Europe, North America, and Asia is at a similar level during the normal period and during the turbulence in the insurance markets. Such information is important for decision-makers who shape the macro-prudential policy of the European insurance sector; in particular regarding the method of determining insurers of global systemic importance. From the studies conducted upon the eight insurers, a hypothesis follows that the level of SR generation increases in the second state in the period from October 17, 2008 to June 5, 2009 shown in Figure 7. A significantly higher level of SR in the state of turbulence means that during turmoil on financial markets, the strength of the negative impact of individual insurance units upon the whole insurance sector increases.

Table 4. Univariate ARMA(1, 1)- eGARCH(2, 2) model estimations for the STOXX 600 Europe Insurance index

Parameter	M	φ_1	ϑ_1	Ω	α_1	α_2
estimation	0.00086	0.68392	-0.72741	-0.20234	-0.25814	0.16812
p-Value	0.35844	0.00000	0.00000	0.00221	0.00003	0.00570
Parameter	β_1	β_2	γ_1	γ_2	ξ (skew.)	ν (shape)
estimation	1.00000	-0.02848	0.09708	0.05358	0.79261	9.87665
p-Value	0.00000	0.00691	0.35870	0.60707	0.00000	0.00438

Source: own study.

Table 5. Bivariate DCC(1, 1) models estimations for the pairs: STOXX 600 Europe Insurance and a given insurer

Indicator	AXA	Allianz	Prud	Legal	Aviva	Metlife	Manu	Ping
c_1	0.02513	0.02159	0.03199	0.04218	0.02631	0.07105	0.03338	0.00942
	0.04014	0.01083	0.02421	0.00990	0.00191	0.02998	0.08039	0.73776
d_1	0.95214	0.96262	0.94015	0.92320	0.96805	0.72663	0.90777	0.85545
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00027	0.00000	0.32845
η (shape)	6.85867	11.11860	8.13343	7.99758	6.31898	16.80825	7.86310	15.97758
	0.00026	0.00241	0.00000	0.00012	0.00000	0.10873	0.00122	0.14136

Source: own study.

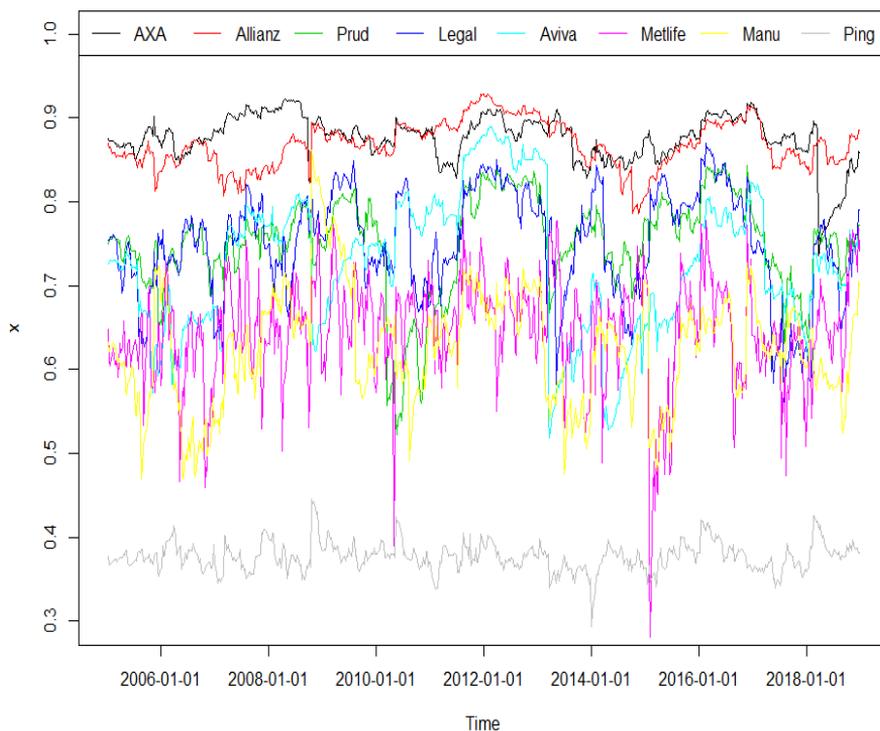


Figure 8. Conditional correlations between the insurer and the European insurance market
Source: own elaboration.

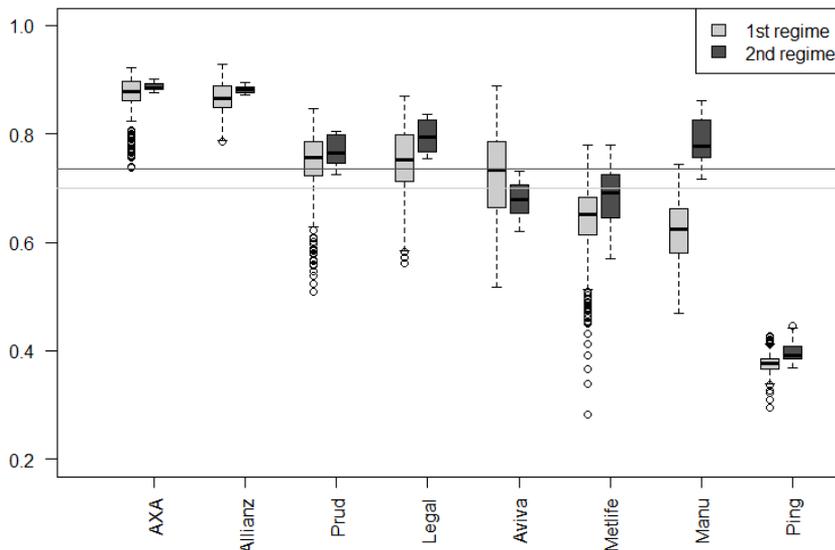


Figure 9. Boxplots for the conditional correlations between the insurer and the European insurance market in the identified regimes

Source: own elaboration.

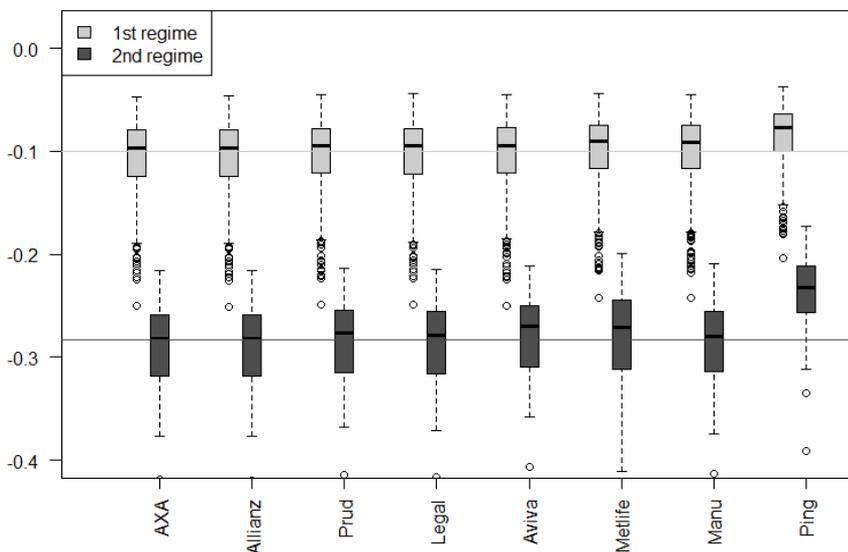


Figure 10. Boxplots for the CoVaR measure in the identified regimes

Source: own elaboration.

Market regimes were established to check whether the systemic importance of the surveyed largest insurers from Europe, North America, and Asia is at a similar level during the normal period and during the turbulence in the insurance markets. Such information

is important for decision-makers who shape the macro-prudential policy of the European insurance sector; in particular regarding the method of determining insurers of global systemic importance. From the studies conducted upon the eight insurers, a hypothesis follows that the level of SR generation increases in the second state in the period from October 17, 2008 to June 5, 2009 shown in Figure 7. A significantly higher level of SR in the state of turbulence means that during turmoil on financial markets, the strength of the negative impact of individual insurance units upon the whole insurance sector increases.

CONCLUSIONS

In this article, we used the C-DCC-GARCH model to analyse dependences in a group formed by the largest five insurance companies from Europe and the biggest insurers from the USA, Canada, and China. Then, availing ourselves of the *CoVaR* measure, we studied the influence of each insurer on the European insurance market systemic risk. The European market was represented by the STOXX 600 Europe Insurance index, while for the insurers, we considered their quotations on domestic markets. The study was performed in two steps. The first one consisted in identifying regimes of European insurance market, while the second one analysed the following items for the identified regimes: correlations among the scrutinised insurance companies (using conditional correlations), dependences between a given insurer and the European insurance market, and the influence of analysed insurance companies on the European insurance market systemic risk. The market regimes were identified by monitoring the insurers' logarithmic returns on shares. To this end, we applied statistical clustering methods for weekly periods to which we assigned the conditional variances obtained from the estimated octovariate c-DCC-GARCH model. Both the clustering quality measures and the possibility of a reasonable economic interpretation exposed two different market regimes in the considered period of time: a regime of low volatility (1st regime, 'normal') and a regime of unstable quotations (2nd regime, 'risky'), which appeared during the time of the strongest turbulences experienced by the global markets.

We may draw the following conclusions from our study:

- The insurance companies from the investigated group are positively correlated. The strongest dependence appears among insurers from Europe – Axa and Allianz are a pair with the strongest tie – a somewhat weaker dependence exists between insurers from Europe and those from North America, while the weakest link shows between the insurer from China and the others. These correlations are clearly stronger in the second identified regime, i.e. during the period of turbulences on global markets (cf. Figure 7). On that basis, we may state that during a global crisis, the exposure to systemic risk on the European insurance market increases, because the stronger the link between insurance companies, the greater the likelihood of the spread of negative effects of financial shocks.
- The European insurance market – as represented by the STOXX 600 Europe Insurance index – is most strongly correlated to the largest insurance companies from Europe, i.e. Axa or Allianz. A weaker correlation exists in the case of insurers from North America and a notably weaker still in the case of the insurer from China (cf. Figure 8). As earlier, these correlations are stronger in the second market regime (cf. Figure 9). Noteworthy, these results may be biased to some extent by the construction of the STOXX 600 Europe Insurance index.

- There is an important difference between the *CoVaR* measures for the first and second regimes of the European insurance market in the case of all the insurers from the studied group. The influence of insurance companies on systemic risk is much stronger during turbulence periods (cf. Figure 10). It is also apparent that in a fixed regime this influence remains more or less at the same level, which in the case of the insurer from China is somewhat lower than average.
- The influence of North American insurance companies on the European insurance market's systemic risk is at a comparative level with the influence of companies from Europe, both in the first and second identified market regimes.

The world entered the twenty-first century, the era of digital economy and the so-called Fourth Industrial Revolution. According to the G20 report, digital economy is defined by economic activity in which digitised information and knowledge are considered to be key production factors, together with the development of a modern information network that accelerates growth and optimises economic structures. The International Monetary Fund (IMF) broadly defines digital economy as digitization in all sectors of the economy.

In the digital age of Fintech, the combination of finance and technology plays an increasingly important role. The financial supervisory and macro-prudential authorities for Systemically Important Financial Institutions (SIFI) now face new challenges, while the prevention of SR is one of the most important elements of globalised policy and economics. Since the reports of the Financial Stability Board (FSB), International Association of Insurance Supervisors (IAIS), and the European Systemic Risk Board (ESRB) keep indicating the lack of tools to describe and measure SR in the insurance sector, further research should concentrate on the search for SR measure. Moreover, scholars should focus on determining the mathematical, statistical, and econometric tools in order to build models that would allow the prediction of adverse phenomena on the insurance market.

The present article uses statistical-econometric tools, a combination of taxonomic methods with the C-DCC-GARCH model, to utilise them in a more widely planned research on SR in the insurance sector when constructing hybrid models using e.g. network theory, as in Denkowska and Wanat (2020).

The article presents several new results, yet it also has some limitations that can be improved in further studies. Based on the presented results in the form of boxplots, we noticed that the overall correlation between companies is higher during turbulence periods. Stronger connections make the whole system more vulnerable to systemic risk, hence the conclusion about the increase in exposure to this risk. However, we did not investigate whether average correlations were significantly different in distinguished states. Nevertheless, this thread of analysis can inspire future studies in which the significance of network connections for systemic risk in the insurance sector is analysed using Minimum Spanning Trees.

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Acknowledgements and Financial Disclosure

This article was financed by the Ministry of Science and Higher Education of the Republic of Poland as part of a research program “Regional Initiative of Excellence” Programme for 2019–2022. Project no.: 021/RID/2018/19. Total financing: 11 897 131.40 PLN.

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