

Assessing the influence of digitalisation on systemic risk in the insurance sectors of European Union countries

Anna Denkowska, Stanisław Wanat, Joao Paulo Vieito

ABSTRACT

Objective: The article aims to study how much digitalisation influences the systemic risk (SR) in the insurance sector of European Union (EU) countries.

Research Design & Methods: In this research, we introduce an innovative, quantitative method for exploring the impact of digitalisation and assessing the similarities and interconnections among all European Union countries' insurance sectors from 2004 to 2018 within the framework of Industry 4.0. The study integrates statistical and econometric tools with network modelling techniques, focusing on the topological indicators of minimum spanning trees (MST) derived from multidimensional dynamic time warping (DTW) distances. We analysed two datasets. The first one comprises exclusively data describing insurance sectors, while the second incorporates data detailing both insurance sectors and the digitalisation processes of individual EU countries. We assessed the similarity of the sectors' dynamics over the analysed period, examining network behaviour during subprime crises, periods of excessive public debt, and immigration-related crises in Europe.

Findings: The proposed tools made it possible to determine how digitalisation contributes to the increase in systemic risk in the EU insurance sector over the periods examined and effectively measure similarity levels, and outline indirect connections between insurance sectors.

Implications & Recommendations: Because similarity can be a potential indirect channel for systemic risk contagion, countries with comparable insurance sectors and shared digitalisation-related behaviours may undergo similar repercussions during global financial downturns. Research endeavours in the insurance sector must consider digitalisation indicators that encompass technological advancements and consumer behaviour.

Contribution & Value Added: We developed a new method to examine the similarity of the insurance sectors of the European Union countries and to assess the dynamics of changes in this similarity in the era of Insurance 4.0. Such an analysis allows for a long-term assessment of the possibility of spreading threats in the insurance sector throughout the European Union.

Article type: research article

Keywords: digitalisation; dynamic time warping; minimum spanning trees; systemic risk; insurance sector

JEL codes: G22, C38, C32

Received: 3 February 2025

Revised: 20 July 2025

Accepted: 9 September 2025

Suggested citation:

Denkowska, A., Wanat, S., & Vieito, J.P. (2025). Assessing the influence of digitalisation on systemic risk in the insurance sectors of European Union countries. *Entrepreneurial Business and Economics Review*, 13(4), 49-69. <https://doi.org/10.15678/EBER.2025.130403>

INTRODUCTION

The prevalent functional classification of financial institutions typically divides them into investment, deposit, and risk-dispersing institutions. The latter group includes entities such as insurance companies, reinsurance companies, life insurance firms, and those offering financial security for old age. Recognising the paramount importance of the insurance sector in economics and finance, and 'given the diversity of initiatives and the high speed of developments, monitoring of digitalisation initiatives in the European insurance sector and assessing opportunities and risks have become an increasingly important priority'

(EIOPA, 2024). Therefore, there is a compelling need for a thorough examination within the framework of the fourth industrial revolution, referred to as Insurance 4.0. Although the World Economic Forum in 2018 (Schwab & Davis, 2018) was dedicated to the Fourth Industrial Revolution, participants gave relatively little specific attention to the insurance sector itself. On the other hand, in IMF (2020) Staff Discussion Notes (SDNs) showcase policy-related analysis and research being developed by IMF staff members and are published to elicit comments and to encourage debate. This document emphasises that due to technological progress and the ever-increasing possibilities of good use of this advanced development, the threat of cyber risk is growing, which is becoming a cross-border and therefore systemic problem. The authors of the document 'Cyber Risk and Financial Stability: It's a Small World After All' present the results of the discussion on this phenomenon. And although they do not devote special attention to the insurance sector, they do indicate the need to subject it to additional financial supervision due to its important position in the entire system.

In Nicoletti's work (2020), the rationale behind and how Insurance 4.0 can holistically revolutionise the insurance industry are explored. Nicoletti (2020) affirms that, thus far, insurance has largely remained on the periphery of the digital transformation wave. However, the global recession and the pandemic have inflicted severe blows on this industry. Post-pandemic, it becomes imperative to devise and execute innovative strategies within the insurance domain. Already, significant segments of the insurance industry are witnessing substantial impacts from digitalisation, particularly in the realm of distribution, which is undergoing a profound digitisation process. Digitalisation is poised to bolster other facets of the insurance industry's value network as well, with customers eagerly anticipating the advent of Insurance 4.0.

Ronken (2018) delves into the concept of Industry 4.0 and its significance for the insurance sector. Insurance 4.0 entails the seamless integration of insurance into a symbiotic ecosystem that fosters exchange-to-exchange (E2E) interactions, where information exchange benefits all participants. This interconnectedness not only expedites the evolution of the insurance sector but also introduces the risk of heightened susceptibility to systemic disruptions and the emergence of novel global risks.

To effectively navigate these challenges and bolster resilience, it is imperative to conduct research aimed at understanding, measuring, and predicting the evolution of interdependencies between various risk types, thereby necessitating the introduction of new concepts to traditional risk management frameworks.

The present article adopts a unique approach by analysing data pertaining to the insurance sector and the levels of digitalisation across the 28 EU countries, employing network theory and the multidimensional DTW algorithm. It addresses a notable gap in the existing literature by exploring the dynamics of similarity among insurance sectors across countries following the EU enlargement.

The next two sections delve into the insurance sector within the context of Industry 4.0, systemic risks within the insurance industry, and present the data used in the analysis. Then, in the following section, we outline the methodology, featuring DTW and MST, elucidating MST topological indicators and introduce a novel approach to MST construction. The next section presents the results, analysing the interconnections between countries, thereby shedding light on potential pathways for the transmission of financial issues. The study concludes by offering insights into the ramifications of digitalisation on the insurance sectors of EU countries.

LITERATURE REVIEW

In this section, we present the literature review concerning phenomena connecting the fourth industrial revolution with systemic risk in the insurance sector, and we motivate the use of the DTW algorithm.

Fourth Industrial Revolution in the Insurance Sector

Technological advancements are fundamentally reshaping production methods and driving economic growth. The initial industrial revolution, which sparked in the eighteenth century, harnessed steam power and mechanised production. Subsequently, the second industrial revolution emerged in the nineteenth century with the introduction of electricity and the assembly line. The third industrial revolution, beginning in the 1970s, brought about partial production automation through programmable memory

controllers and computers. According to Hilbert (2020), in the late 1980s, less than 1% of data was digital, while by 2012, 99% of the world's information had become digital. Presently, we find ourselves amidst the fourth industrial revolution, commonly referred to as 'Industry 4.0,' characterised by the integration of information and communication technologies into industrial operations. We may find an example in the burgeoning LendTech sector within FinTech (Kaur & Singh, 2023; Waliszewski *et al.*, 2024).

This transformative phase, driven by artificial intelligence (AI), the Internet of Things (IoT), robotics, and big data, is reshaping global economic systems, including the insurance sector. However, Lyons (2019) highlights the insurance sector's limited or slowed involvement in the fourth industrial revolution. Simultaneously, it underscores the immense potential for transforming insurance into Insurance 4.0. Masiello (2020), in the book 'Insurance Agency 4.0: Prepare Your Agency for the Future; Develop Your Road Map for Digitalisation; Increase Profit, Scalability and Time,' gathers solutions and strategies for insurance agents and the sector to support the development of Insurance 4.0. Eling and Lehmann (2018) argue that while other industries have already undergone significant transformation through the integration of analogue and digital worlds with new technologies, the insurance industry's transformation has been relatively slow, with digital technologies yet to be fully exploited. They support their argument by analysing 84 scientific articles and creating a database of research, articles, and working papers assessing the impact of digitalisation on the insurance sector, focusing on business and economic literature on risk and insurance. From their analysis, we can conclude that the relationship between new technologies and insurance will raise numerous new research questions.

Industry 4.0 brings forth many new issues and questions for the insurance sector. Participants in the Swiss Risk and Insurance Forum in 2016, 2018, and 2019 discuss the digitalisation of the insurance sector, summarising the main topics and findings from these forums (Albrecher *et al.*, 2016; Albrecher *et al.*, 2018; Albrecher *et al.*, 2019). The events gathered experts from the insurance industry, regulatory bodies, and consulting firms to discuss the challenges arising from the impact of data science and digitalisation in the insurance sector. Eling and Lehmann (2018) assert that the academic community should also be part of the discussion on how to utilise digital technologies in the economy and society.

In a comprehensive analysis, as presented in the paper (Denkowska *et al.*, 2023), the authors observe notable disparities in the insurance markets of EU countries. Factors such as a nation's economic development, household wealth, and insurance premiums significantly influence the structure of its insurance market. Building upon the research by Denkowska *et al.* (2023), we continue to investigate the impact of digitalisation on the insurance sector of EU countries and its dynamics. To evaluate the dynamics of changes and their impact on development in these countries, we employed the multidimensional DTW algorithm, assessing classic measures describing the insurance sector. These measures include insurance market penetration, insurance market density, insurance market concentration, average value of total insurance assets, investment to GDP ratio, and the number of companies in the total market per 1 million inhabitants. To examine the quantitative impact of digitalisation on the EU countries' insurance sector, we utilise appropriate indicators to gauge digitalisation levels in various aspects of society. These indicators cover five key areas of digitalisation development: broadband internet and household connectivity, the percentage of households with an internet connection, people with ICT education based on employment status, online activities such as banking and information searches, individual online shopping behaviour, and efforts to promote e-commerce and digital integration among individuals. Our research spans the period of 2004-2018. In the context of the fourth industrial revolution, the Digital Economy and Society Index (DESI), established in 2014, serves as a crucial development indicator, which we considered separately in our research.

Systemic Risk in the Insurance Sector

The insurance market is now an integral component of the comprehensive safety net within the European Union financial market. This inclusion stems from the imposition of financial management rules on insurance companies, the establishment of regulations on the capital requirements of market entities, the introduction of common principles for public and internal supervision, the enforcement of strict procedures for financial services provision, and the extension of consumer safety guarantees. Oversight of the Union's financial security is conducted by the European Banking Authority,

the European Insurance and Occupational Pensions Authority (EIOPA), and the European Securities and Markets Authority.

EIOPA's primary goal is to ensure the effective implementation of legislation in the insurance sector to maintain financial stability and adequately protect consumers of financial services. Financial crises have underscored that SR is a crucial issue in a globalised economy. Despite historically insurance sector was viewed as a non-generating SR, in-depth research following the AIG crisis in 2008 has revealed the potential for the insurance sector to create or increase SR.

Academic research aims to identify effective predictive methods that support macro-prudential supervision. A key requirement for successful regulation and supervision is the authority's ability to collect and analyse comprehensive insurance market data. According to IAIS, IMF, and NAIC, collected data should offer the opportunity to assess insurance activity and risk based on indicators related to profitability, income generation, capital/provisions and leverage adequacy, liquidity, underwriting performance, risks, investment performance, and reinsurance performance and risks.

Kwon and Wolfrom (2017) discuss the analytical tools used by supervisory authorities for market and macro-prudential purposes in 24 OECD countries and beyond. The indicators described assess the insurance sector, including aspects of validity, competition, scope of activity, insurance market risk, and SR. The article presents information on monitoring indicators related to interconnections and changes in asset allocation – two areas with potential SR implications.

The OECD describes how 24 selected countries undergo Insurance Market Risk analysis with periodic reviews of market risk, key indicators, interconnectedness, changes in asset allocation, and stress testing indicators. A common analytical tool used in many countries involves periodic reviews of specific types of risk for the functioning of insurance markets. In Europe, insurance regulators contribute to the analysis of financial stability by publishing reports prepared by EIOPA every two years. The document by the EIOPA (2017) initiated a series of documents on SR in the insurance sector, supporting macro-prudential policy and contributing to the debate on the specific nature of insurance activity in the context of SR.

The International Association of Insurance Supervisors (IAIS), which aims to promote effective and globally consistent supervision of the insurance industry, brings together insurance supervision authorities from over 130 countries and international entities, including the OECD, World Bank, and International Monetary Fund. In July 2013, nine insurance institutions were identified as Globally Systemically Important Insurers (G-SIIs), marking institutions of particular importance for global financial stability due to their size, market power, and global connections. The criteria for determining systemically important insurance institutions (EIOPA, 2017) include size (5%), global activity (5%), interconnectedness (40%), asset liquidation (45%), and substitutability (5%). Given the substantial weight assigned to interconnectedness, minimum spanning trees (MST) indicators and networks between countries are utilised to analyse the potential generation and spread of SR.

Regulation (EU) 2019/2176 of the European Parliament and the Council, amending Regulation (EU) No 1092/2010 outlines the determination of systemic risk. It establishes a European Systemic Risk Board and defines SR as undesirable events with systemic importance, affecting the financial system and the real economy. The reasons for the implementation of these events are systemically important, with general and specific reasons mainly related to the financial sector. The consequences of these events impact the real economy. The need to analyse systemic risk is highlighted in many studies on risk management (Jajuga, 2023).

We may find various concepts of systemic risk analysis in Harrington, 2009; Adrian and Brunnermeier, 2016; Jajuga *et al.*, 2017; Bisias *et al.*, 2012. Moreover, we find an approach based on different measures, *e.g.*, conditional value at risk (CoVaR) and Δ CoVaR (Adrian & Brunnermeier, 2016), Co-Risk (Chan-Lau, 2010), marginal expected shortfall (MES) (Acharya *et al.*, 2017), dynamic causality index (DCI) (Billio *et al.*, 2012), SRISK (Brownlees & Engle, 2017), systemic contingent claims analysis (Gray & Jobst, 2013). In recent years, apart from the measurement methods presented above, network analysis has become the basic tool for examining systemic risk where connections are established using different methods: Granger causality networks (Billio *et al.*, 2012), vector autoregression model (variance decomposition (Diebold & Yilmaz, 2014)), information theory (transfer entropy (Bekiros *et al.*, 2017)). Relevant associations are captured by, *e.g.*, minimum spanning trees; glasso, tlasso methods

(Torri *et al.*, 2018) and using topological network indicators derived from graph theory (Denkowska & Wanat, 2019; 2020a; 2020b; 2021; 2022).

We analysed the indirect links of institutions within the insurance sectors of EU countries, expressed by the similarity in the areas of insurance markets, considering digitalisation indicators. We assumed that in groups of countries with similar insurance sectors exhibiting comparable behaviours, economic conditions (*e.g.*, GDP), and a similar level of internet access and usage, we would observe similar behaviours in the event of a crisis. Hence, we encoded the SR transmission channel between sectors of EU countries in the apparent similarity of these sectors. The literature does not typically analyse the global (entire insurance sector) approach to SR. We analysed the entire insurance sector of EU countries based on data describing entire sectors and not individual insurers. The proposed approach in this article is innovative. The novelty is the new construction of MST based on the multivariate DTW algorithm. We present this construction in the methodology section.

Dynamic Time Warping Motivation

Dynamic time warping stands as one of the algorithms designed for assessing the similarity between two time series of varying lengths, even those that may exhibit temporal differences. Originally introduced in the 1960s by Bellman and Calaba (1959), DTW has gained prominence in recent years, emerging as the preferred distance measure for various time series data mining applications. Widely utilized in non-economic fields such as speech recognition (Sakoe & Chiba, 1978), image processing (Cedras & Shah, 1995), motion recognition (Geiger *et al.*, 1995), ECG analysis, biometrics, signal analysis, and data mining (Keogh & Pazzani, 2000), DTW has demonstrated its versatility.

In a study by Petitjean *et al.* (2011), the non-parametric DTW measure of similarity outperformed other measures like the Pearson correlation coefficient. However, despite its potential, Franses and Wiemann (2020) assert that economic research has not fully harnessed the capabilities of dynamic time warping. Notable exceptions include the study of Wang *et al.* (2012), representing some of the few instances where the DTW technique has been applied in economic research.

Noteworthy, the DTW algorithm goes beyond mere distance calculation. It captures the underlying similarity between time series. In our research, we used the multidimensional DTW algorithm. We used this measure of similarity of multivariate time series here as the value assigned to the edges between MST vertices that correspond to the insurance sectors of EU countries.

To sum up this introduction to the examined issue, we would like to emphasise that our contribution to the development of the methodology and our empirical research fills a research gap. The proposed methodology allows for a multidimensional analysis of the insurance sectors of EU countries. We will empirically confirm that the topological indicators of innovatively constructed MSTs allow for the assessment of systemic risk in the European Union insurance sector. The study provides evidence that the analysis of systemic risk in the insurance sector should take into account society's activities related to digitalisation. The dynamics of the structure of connections between the insurance sectors of EU countries change if we consider the processes of the Industrial Revolution 4.0. The effects of the Revolution 4.0 may increase threats in the entire insurance system.

RESEARCH METHODOLOGY

In this section, we present the data and tools used in the research. We considered two datasets: the first one characterises the insurance sector, while the second one describes consumers' behaviour in the context of digitalisation. We use tools such as the MST based on three types of weights. The first weight is determined using the Mahalanobis distance, while the other two are derived from the DTW algorithm applied in two different ways. The main goal of this research was to analyse the dynamics of the MSTs structure. We followed topological indicators of the MSTs to observe when and how potential risks may propagate during the period 2004-2018, focusing on different crisis subperiods within this timeframe.

Dynamic Time Warping Algorithm and Minimum Spanning Tree

In this part, we present the tools used in the research. We discuss the mechanism of the DTW algorithm, the construction of MSTs, as well as the indicators describing the structure of these MSTs and their interpretation in the context of systemic risk (Denkowska & Wanat, 2022).

We considered two time series $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_m)$ for $n \in \{1, 2, \dots, N\}$, $m \in \{1, 2, \dots, M\}$, and $N, M \in \mathbf{N}$.

We denoted by S the space of the series elements that we compare

Therefore $x_n, y_m \in S$ for $n \in \{1, 2, \dots, N\}$ and $m \in \{1, 2, \dots, M\}$.

To compare different features, you need a local measure of costs, sometimes also known as a local distance measure, i.e. a function $c: S \times S \rightarrow \mathbf{R}_+$. Usually, $c(x, y)$ is small (low cost), if x and y are similar; otherwise $c(x, y)$ is large (high cost). Determining the local cost measure for each pair of terms of X and Y series, we obtained a cost matrix $C_{N \times M}$ defined by $C(n, m) := c(x_n, y_m)$.

The local cost measure can be determined by e.g., $c(x_n, y_m) = |x_n - y_m|$ or $c(x_n, y_m) = (x_n - y_m)^2$.

Then the warping path is determined. A warping path is a sequence $w = (w_1, \dots, w_K)$, for $w_k = (n_k, m_k) \in \{1, \dots, N\} \times \{1, \dots, M\}$, for $k \in \{1, \dots, K\}$ and meets the following conditions:

- (i) Boundary condition: $w_1 = (1, 1)$ and $w_K = (N, M)$
- (ii) Monotonicity condition: $n_1 \leq n_2 \leq \dots \leq n_K$ and $m_1 \leq m_2 \leq \dots \leq m_K$
- (iii) Step size condition: $w_{k+1} - w_k \in \{(1, 0), (0, 1), (1, 1)\}$ for $k \in \{1, \dots, K-1\}$.

Note that the step size condition (iii) implies the monotonicity condition (ii), which is nevertheless explicitly cited for the sake of clarity. The warping path $w = (w_1, \dots, w_K)$ defines the alignment between two series $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_m)$ by assigning an item x_n from X to the item y_n from Y . The boundary condition (i) forces the first elements of X and Y respectively and the last elements of X and Y to be aligned respectively with one another. In other words, the alignment applies to the entirety of the X and Y sequences. The monotonicity condition reflects the requirement for faithful timing: if an element in X precedes the other, this should also be maintained for the corresponding elements in Y and vice versa. Finally, the step size condition expresses a kind of continuity condition: no element in X and Y can be omitted, and there are no repetitions in the alignment (in the sense that all the pairs of indices included in the warp path p are pairwise distinct).

The total cost $c_w(X, Y)$ of the *warping path* w between X and Y with respect to the local cost measure $c(x, y)$ is defined as $c_w(X, Y) := \sum_{k=1}^K c(x_{n_k}, y_{m_k})$

An optimal warping path between X and Y is denoted by w^* – it is a warping path with the minimum total cost of all possible paths.

$DTW(X, Y) := c_{w^*}(X, Y) = \min\{c_w(X, Y), \text{ where } w \text{ is an } (N, M) \text{ warping path}\}$.

Most studies in the literature consider only the one-dimensional case. The generalisation of DTW to the multivariate case is typically approached in one of two ways: dependent or independent warping. An algorithm that differentiates between data types and adapts the computation of multivariate DTW is discussed in Shokoohi-Yekta *et al.* (2017). Machine learning mechanisms can serve in the proposed approach, as described by Zhao *et al.* (2016), to learn multiple local Mahalanobis distance metrics for k-nearest neighbour (kNN) classification of temporal sequences.

Identifying, measuring, and predicting systemic risk within the financial sector necessitates employing tools that facilitate the assessment and quantitative analysis of the entire system. The interconnectedness pattern among the elements within the system plays a pivotal role in understanding its behaviour (Boccaro, 2003). Indeed, one cannot fully comprehend a system by isolating its individual components (Andrzejak *et al.*, 2024). A widely adopted approach to scrutinising interaction patterns within a given system involves constructing a network (graph) comprising nodes and edges (Albert, 2002). Thus, we employed network theory in our research.

The MST utilised in our study, introduced by Kruskal (1956), encapsulates information about the global structure of the network, simplifying analysis by capturing the most significant connections among the studied entities. A minimum spanning tree is a connected and acyclic graph with the least

sum of weights assigned to each edge. Numerous recent studies in economics and finance leverage MSTs to investigate the topological structure of these networks. For instance, Andrzejak *et al.* (2024) analysed the currency network using various methods to determine the distance between nodes and compare the topological structures of the resulting networks. They identified a method that effectively describes the currency market's dependence structure, particularly sensitive to changes.

In our case, the vertices of MST are the insurance sectors of individual EU countries, and we assign a distance to each edge, as an innovation, we shall do this in three different ways (see the next point). The purpose of the modification of the MST structure, which was originally proposed by Mantegna (1999), is to use MST indicators to analyse the similarity dynamics of EU insurance sectors. The edges with the lowest weights are interpreted as relatively short distances and hence show great similarities between the pairs of EU insurance sectors.

Topological MST Indicators and Their Interpretation in the Context of Systemic Risk

We analysed the time series of selected MST topological indicators such as average path length (APL), maximum degree (Max.Degree), parameters 'alpha' of the power law of the degree distribution (alpha), network diameter (Diameter), assortativity.

Average path length (Wang *et al.*, 2014). This indicator is defined as the average number of steps along the shortest paths for all possible pairs of network nodes (vertices). It measures the effectiveness of information flow or mass transport in a given network. APL is one of the strongest measures of network topology, along with its grouping factor and degree distribution. It distinguishes an easy-to-go network from a more complex and inefficient one. The smaller the average path length, the easier the flow of information. Of course, we are talking about average, so the network itself can have several very distant nodes and many adjacent nodes.

Maximum degree (Wang *et al.*, 2014). This is the number of edges connected to a given vertex. If the maximum degree is growing, it means that in the group of insurers, some insurer has many more connections with others. In a situation of shock, in such a vertex, the risk of spreading its effects increases.

Parameter α of the vertex degree distribution required to follow a power law (Wang *et al.*, 2014). This indicator measures the scale-free behaviour of a network (Rak & Rak, 2020). The network is called scale-free if the distribution $P(s)$ of the number of links between the vertices follows a power law, i.e. it has (asymptotically) the form $P(s) = C \cdot s^{-\alpha}$, $\alpha > 0$, where α is a parameter specific to the given network. The power law followed by the degree distribution gives the network a kind of fractal self-similarity properties, which explains the name. A scale-free network is characterised by a small number of vertices having a large number of connections (such nodes are called hubs) and many vertices that have only one connection. Such a network is 'favourable' to the propagation of systemic risk, and the companies-hubs that it has are systemically relevant. If the MSTs are scale-free, with the alpha value being closer to 2, it means that the MST structure is star-shaped, the hubs are high-degree nodes.

Diameter of the network (Li *et al.*, 2018). It is determined by choosing from among all the shortest paths connecting any pair of vertices the longest one. For MST, this is simply the longest path in the MST. When the diameter decreases, it means that the farther lying nodes become closer.

Assortativity (Newman, 2002) is a graphic measure. It shows to what extent nodes in the network associate with one another by similarity or opposition (positive or negative mating). Basically, the network's assortativity is determined by the degree (number of direct neighbours) of nodes in the network. Assortativity is expressed as a scalar $-1 \leq \rho \leq 1$. The network is said to be assortative when high-degree nodes are mostly connected to other high-degree nodes, while low-degree nodes are mostly connected to other low-degree nodes. The network is said to be non-assortative when high-degree nodes are connected mostly to low-degree nodes and low-degree nodes are mostly connected to high-degree nodes. Assortativity provides information on the structure of the network, but also on its dynamic behaviour and robustness. The original definition of assortativity (Newman, 2002) for unweighted, undirected networks is based on the correlation between random variables. A negative assortativity means that in each state the tree is rather non-assortative, i.e., the vertices tend to merge rather on a less similar basis: those that have a high degree with those that have few connections. This is usually associated with the previously described property of a scale-free network.

MST Construction

In this part, we present how the MST is constructed in three different ways. The proposed methods allow for capturing the dominant structural patterns in the analysed time intervals. By analysing the topological indicators of each MST, which describe their structure, we assess the dynamics of changes in the interconnections between EU insurance sectors. To fully reflect the dynamic relationships, MSTs are constructed in three different ways, also using a rolling window approach. The first method assigns weights to the edges based on the standard Mahalanobis distance, calculated year by year between pairs of EU insurance sectors. The second method uses a multidimensional DTW distance as the edge weights. The distance between time series is determined incrementally for each subsequent year. The third method applies a multivariate DTW distance calculated over four-year periods. For each year, we determine the distance between the time series using the data from the corresponding four-year window. For each type of MST construction, we also provide an explanation of how to interpret the information contained in each constructed MST.

MST Based on the Mahalanobis Distance

For fixed multidimensional vectors describing a given country, we considered the Mahalanobis distance, which is the distance between two points in a multidimensional space that differentiates the contribution of individual component coordinates of points and uses the correlations between them.

It is defined as $d_m(x, y) = \sqrt{(x - y)^T C^{-1} (x - y)}$, where $x = [x_1, \dots, x_n]$, $y = [y_1, \dots, y_n]$ are vectors from \mathbb{R}^n and C is a symmetric, positive definite matrix. This distance is commonly referred to as the Euclidean weighted distance, where C^{-1} is the weight matrix. While Euclidean distance measures the straight-line distance between two points, the Mahalanobis distance takes into account correlations and the spread (variance) of the data. This means that if the data is stretched in a certain direction (e.g., due to variable correlation), the Mahalanobis distance will 'shorten' the distance along that direction. Since we aimed to capture similarities between multidimensional datasets, and because the Mahalanobis distance is also robust concerning differences in measurement units and scales, its application is more appropriate for our research than that of the standard Euclidean distance. Further, using Kruskal's algorithm, we construct the MST for each year in the analysed period from 2004 to 2018.

In the next step, we generated a time series of topological indicators that describe the structure of each MST. Based on these indicators, we concluded the interrelationships and similarities between the insurance sectors of EU countries each year. Each value of a topological indicator in a given year reflects the structural characteristics of the MST for that year, thus defining the links between the EU insurance sectors. Based on the topological indicators of the 15 MSTs constructed using data from 2004-2018, we assessed the dynamics of changes in interconnections between the insurance sectors of EU member states and evaluated the significance of these changes for systemic risk contagion across the Union.

MST Based on Multidimensional Distance DTW in an Incremental Manner

In this MST construction, we used vectors from \mathbb{R}^n representing the EU countries for each year in the period 2004-2018. For the given time series, we computed a multidimensional DTW distance for each of the 28 countries in such a way that, for each year from 2004 to 2018, the DTW distance is calculated incrementally. That is, the DTW in the n -th year reflected the similarity of the time series from 2004 up to year n . Based on the DTW distances obtained in this way, we constructed MSTs for each year using Kruskal's algorithm, as in the previous approaches.

The MST corresponding to a given year reflects the structural similarity between EU insurance markets accumulated from 2004 to that year. The evolution of these MSTs and their associated topological indicators reveal how the interlinkages between the insurance sectors of EU countries have changed over time, and whether integration or fragmentation has taken place. Each yearly MST provides insights into the changing dependencies across the EU insurance sector, capturing both current and historical dynamics since 2004.

MST Based on Multidimensional Distance DTW from the Period in Four-Year Windows

In this construction, as in the second approach, we calculated a multidimensional DTW distance for each of the 28 countries based on the given time series. However, for each year in the period 2004-2018, the DTW distance was determined over a rolling four-year window. That is, the DTW value in year n reflected the similarity of the time series over the four years preceding year n . Using these DTW distances, we constructed MSTs for each year, as before, employing Kruskal's algorithm.

The MST for a given year captures, therefore, the structural similarity of EU insurance markets during the preceding four-year period, enabling an assessment of recent trends and short-term interdependencies between the sectors.

RESULTS AND DISCUSSION

In this section, we present the results concerning the evolving structure of MSTs. It is also important to emphasise that we applied the three proposed methods of MST construction to three distinct datasets. As mentioned earlier in the data section, we based the analysis on the following data sources: the first set described the insurance sector from 2004 to 2018, the second set included both the insurance sector and digitalisation indicators for the same period, while the third dataset extended the analysis to include insurance sector data alongside five DESI domains from 2014 to 2018.

We conducted an analysis comparing the development of the insurance sector with its progression, considering additional data that must be factored in during the era of the Fourth Industrial Revolution. Insurance activities are increasingly intertwined with Internet usage and the ongoing development of new technologies, such as real-time information transmission and location-based services. This comprehensive analysis of the intersection between the insurance sector and digitalisation has not been performed in the literature to date.

The time series of MST topological indicators reflects the dynamics of changes in the similarity and interconnections between the insurance sectors of EU countries over the 15 years. Each method of MST construction leads to distinct results. The topological indicators obtained from different approaches contain complementary types of information. The MST based on the Mahalanobis distance captures the relationships between the insurance sectors of EU countries only within a given year.

In contrast, the MSTs constructed using multivariate DTW distances reflect the similarity of development patterns across countries either over the entire period from 2004 to the given year or within rolling four-year windows. This allows us to detect similarity in trends even when countries develop at different rates but follow comparable trajectories. As such, DTW-based MSTs capture the structural convergence of insurance sectors over time. Thus, although in a given year the Mahalanobis distance may show significant differences between countries, the DTW-based approaches may reveal strong long-term or short-term similarities in the development paths of their insurance sectors.

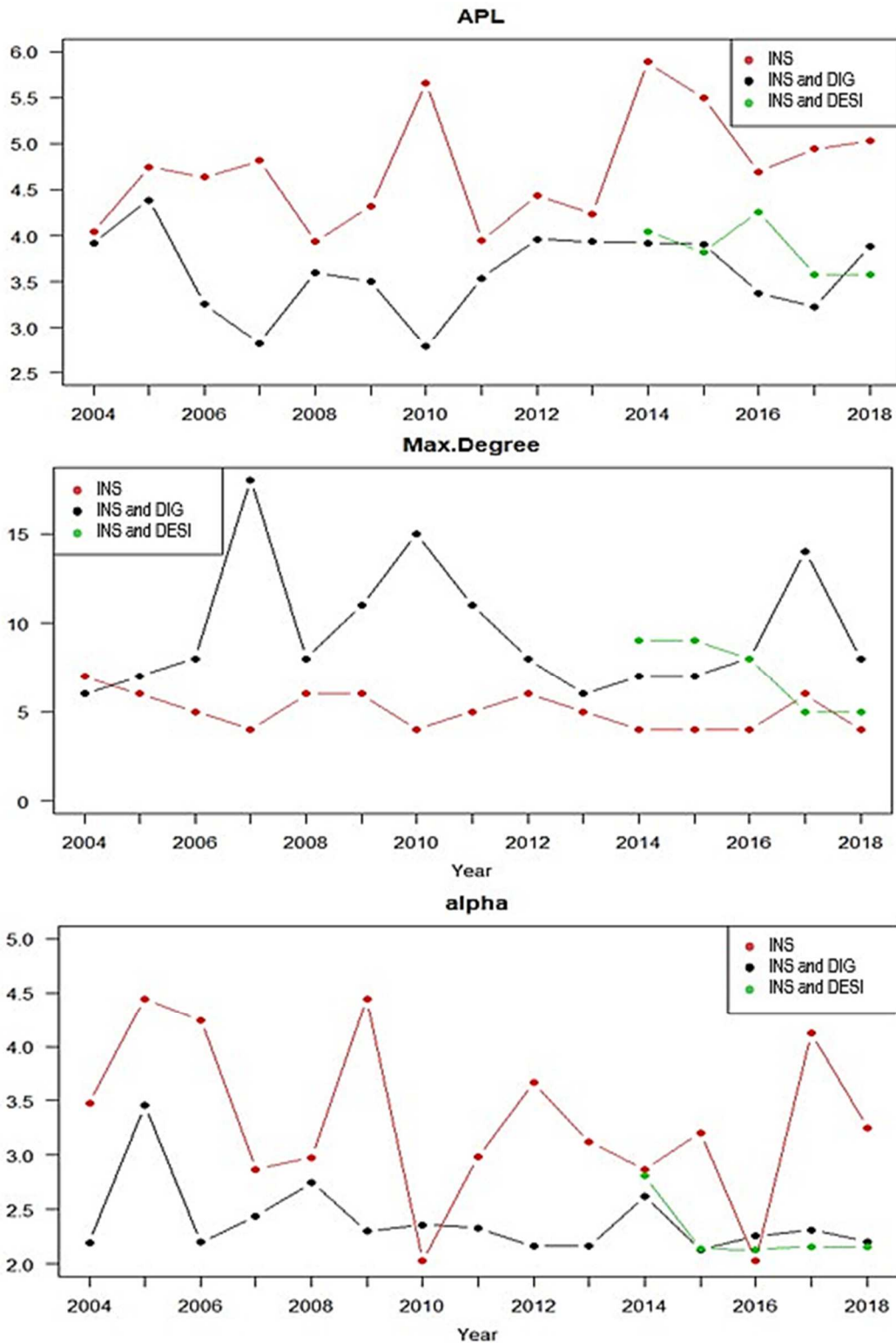
In Figures 1 and 2, we present examples of MSTs, both constructed using data from the year 2007. The MST in Figure 1 illustrates the interconnections between the insurance sectors of EU countries based on the Mahalanobis distance, using only data describing the insurance sector. In contrast, Figure 2 presents an MST constructed using the same method but based on 17 variables – 9 describing the insurance sector and 8 related to the level of digitalisation.

The structures of these MSTs differ significantly. In Figure 1, the MST appears more dispersed and elongated, resembling a chain-like structure. In contrast, the MST in Figure 2 is more centralised, forming a star-like topology. In this structure, Portugal occupies a central position and plays a crucial role, it is the vertex with the highest degree, which is notably high.

If a negative financial event were to occur in the insurance sector of a particular country, and if similarity within the MST acts as a channel for systemic risk transmission, the 2007 MST structure shown in Figure 2 suggests that such an event could propagate rapidly to other countries closely connected to the affected one. This highlights the importance of considering digitalisation in the analysis of systemic risk (SR) within the insurance sector. By comparison, the structure of the MST in Figure 1 appears less concerning in terms of SR contagion.

During the crises themselves, the structure loosens, suggesting that the inhabitants of EU countries exhibit behaviour aimed at shielding themselves from the effects of crises. The red charts show opposite behaviour. Before crises, the MSTs are less concentrated, and during crises, we observe clustering and growing similarity among insurance markets. This is not surprising.

Unfortunately, such a structure of interconnections between countries reinforces the spread of undesirable financial disruptions. Noteworthy, the time series of black-coloured topological indicators may serve as a predictive tool, before each of the analysed crises, these indicators displayed similar patterns.



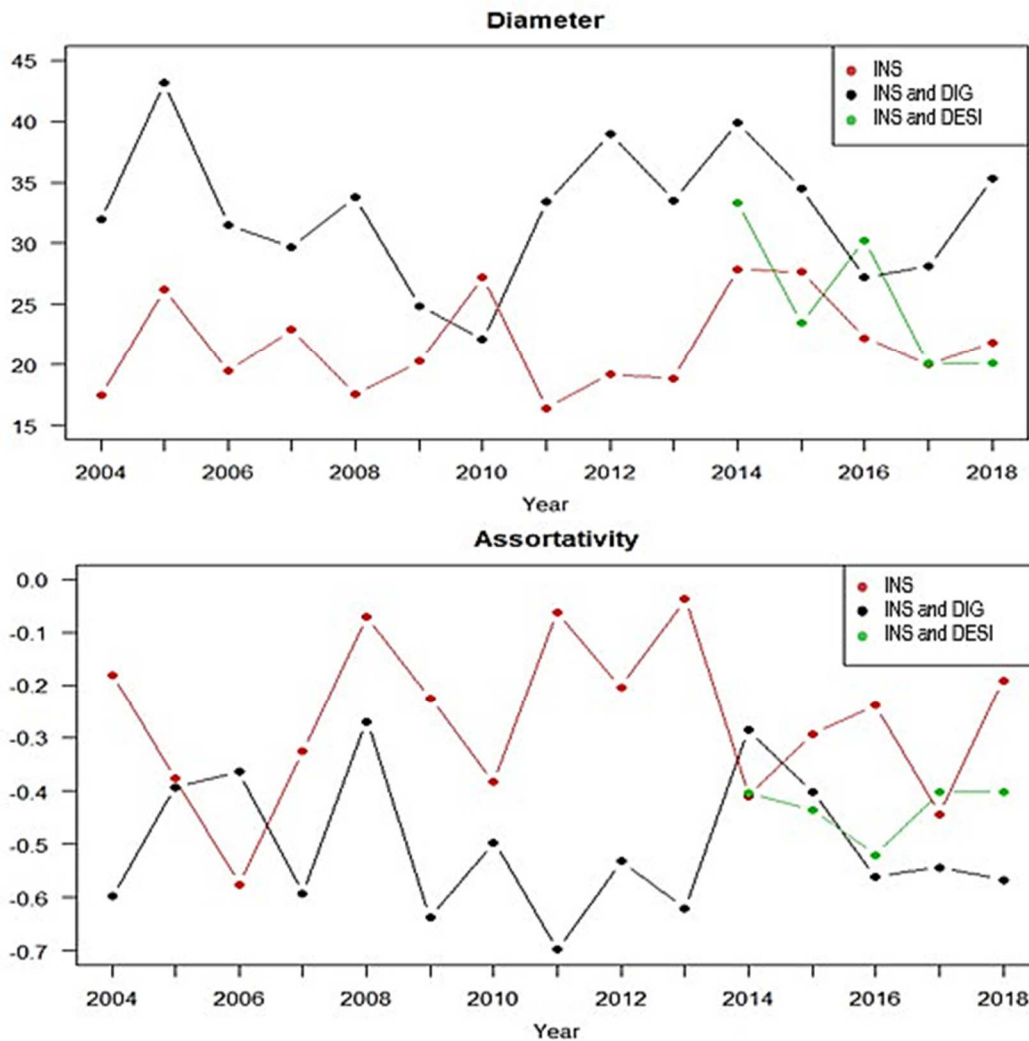


Figure 3. MST indicators for MST based on the Mahalanobis distance

Source: own elaboration in RStudio.

The analysis presented below focuses on the second direction of our research, presented in Figure 4. While the first part, already discussed, provided an empirical investigation of the annual similarity between the insurance sectors of EU countries, this stage aims to assess their similarity over extended periods. Each value of the minimum spanning tree (MST) topological indicators now informs us about how similar the insurance sectors are in the cumulative period from 2004 to a given year. This approach also allowed us to trace how similarity evolves as the dataset expands year by year.

As before, we present the time series of topological indicators with consistent colour coding for the respective datasets. The graphs in Figure 5 show the evolution of MST topological indicators calculated from: (1) nine time series describing the insurance sector (red) and (2) seventeen time series combining insurance sector data with digitalisation metrics (black). These series differ in both behaviour and value ranges.

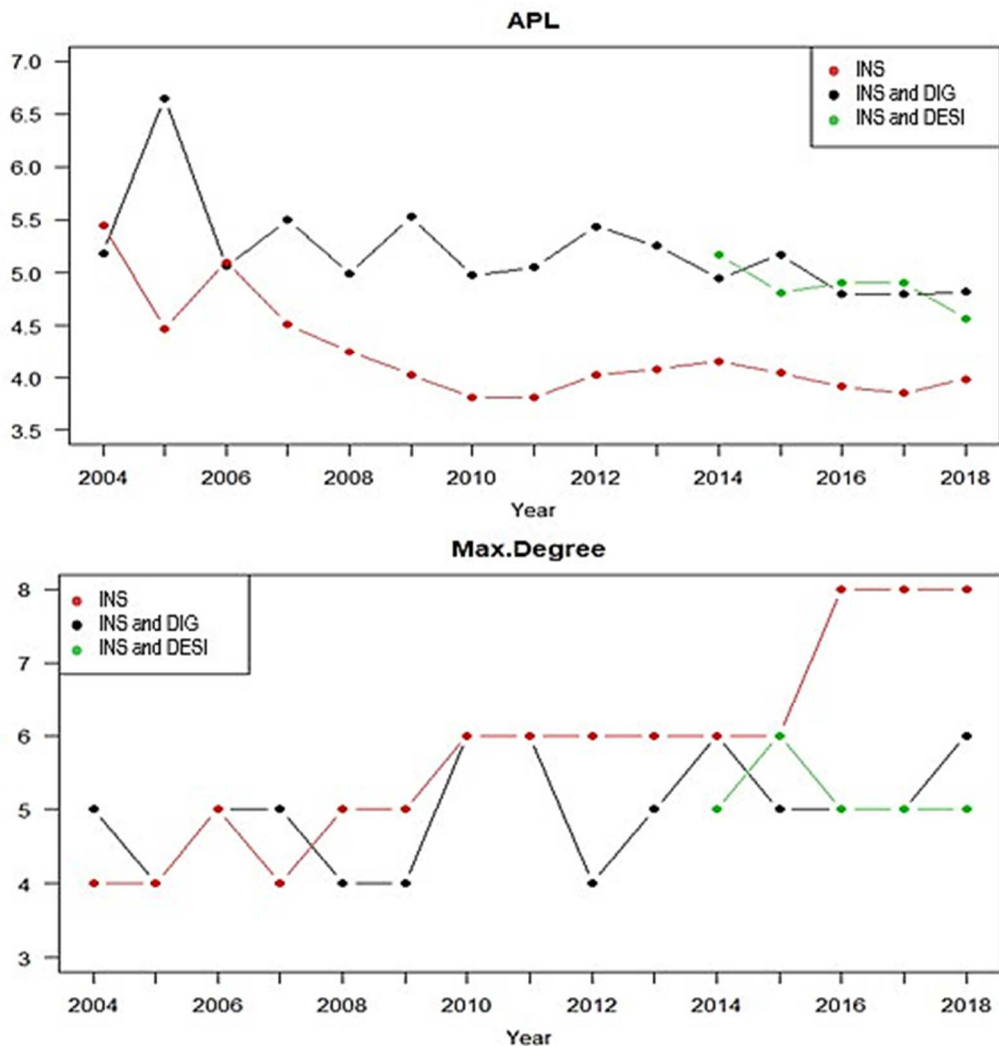
Assessing similarity over extended periods using MSTs derived from multidimensional DTW distances enables the detection of structural changes in the EU insurance sector. It also facilitates the identification of long-term dependencies and similarities between countries. Notably, the black and red series display differences in magnitude. The red series tends to remain stable over time, while the black series shows higher variability.

The fluctuations in the black series suggest that extending the 17-dimensional dataset by a single year, especially one marked by structural change, significantly affects similarity with the previous period. For instance, a decline in APL and Diameter accompanied by a rise in max. degree, as observed in

2008, 2010, 2014, and 2016, points to increasing similarity among EU insurance sectors. The alpha and assortativity indices further illuminate the way sectors (represented as vertices) are connected. The MST structure is irregular and non-fractal, as alpha deviates from 2. There are no dominant hubs, and vertex connections form uneven clusters of varying sizes.

These structural changes are likely driven by the inclusion of digitalisation indicators. For example, in 2008, the black APL chart drops below its 2007 value, indicating that the MST, based on the extended multidimensional time series, reflects lower average path length, and hence greater similarity. This underscores the influence of digitalisation on the evolving similarity structure.

In general, the black APL and diameter values tend to be higher than those of the red series, while max. degree is relatively lower. This combination, high APL and diameter with low max. degree characterises a more stretched MST, suggesting reduced potential for the rapid spread of negative systemic effects. During periods of financial stress, including 2008, 2010, 2011, 2014, and 2016, changes in the black series confirm that digitalisation meaningfully impacts the evolving similarity and interconnectedness of EU insurance sectors.



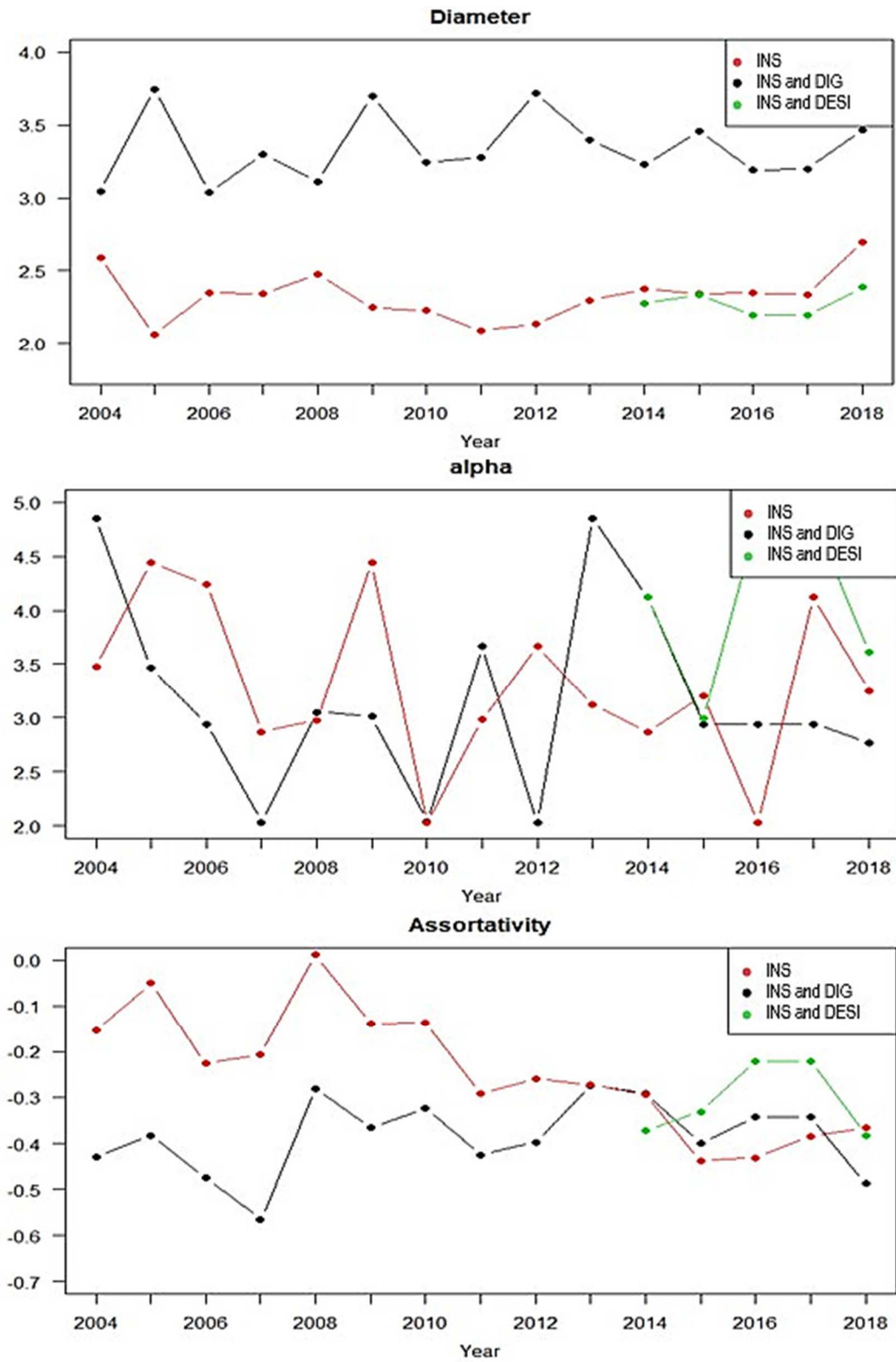


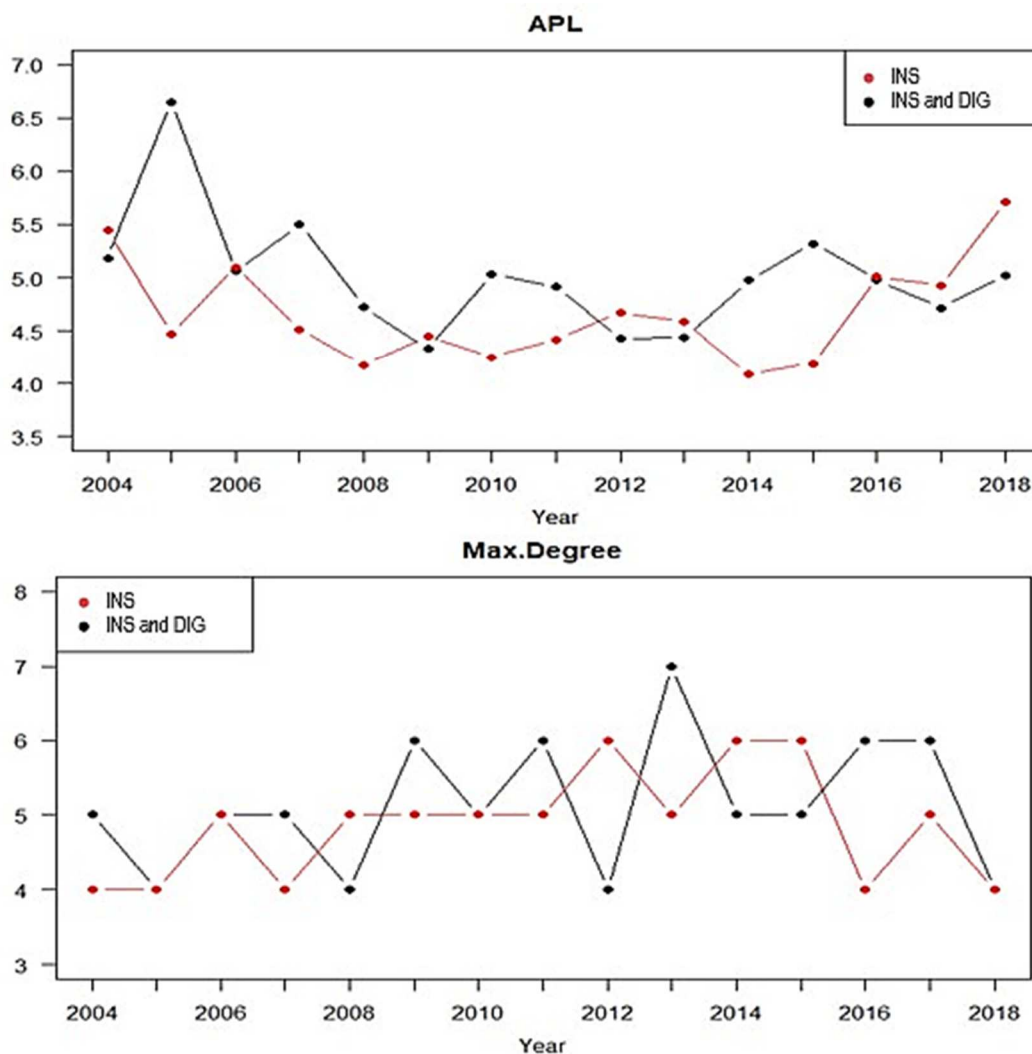
Figure 4. MST indicators for MST based on multidimensional distance DTW in an incremental manner

Source: own elaboration in RStudio.

In the next step, we present the graphs of topological indicators in Figure 5, which describe the structure of the MST constructed using the third method. Each value of the MST indicator reflects the similarity within the groups of countries based on nine time series (red) and seventeen time series (black).

The dynamics of these indicators differ from the previous cases, and both data sets exhibit noticeable volatility.

During the crisis periods of 2007-2009, 2010-2012, and 2015-2017, the black lines representing APL and diameter show a downward trend, while the maximum degree increases. This suggests that, in each four-year window preceding a given year, the MST had a less centralised (less star-like) structure compared to the structure observed in that specific year. The shift toward a more star-like structure indicates an elevated potential for the propagation of negative effects from financial shocks. This shift is particularly significant when considering the role of digitalisation in the insurance sector. Digitalisation intensifies the interdependencies between countries, amplifying the risk of contagion in the face of global financial disruptions, which further increases the systemic risk within the EU's insurance markets.



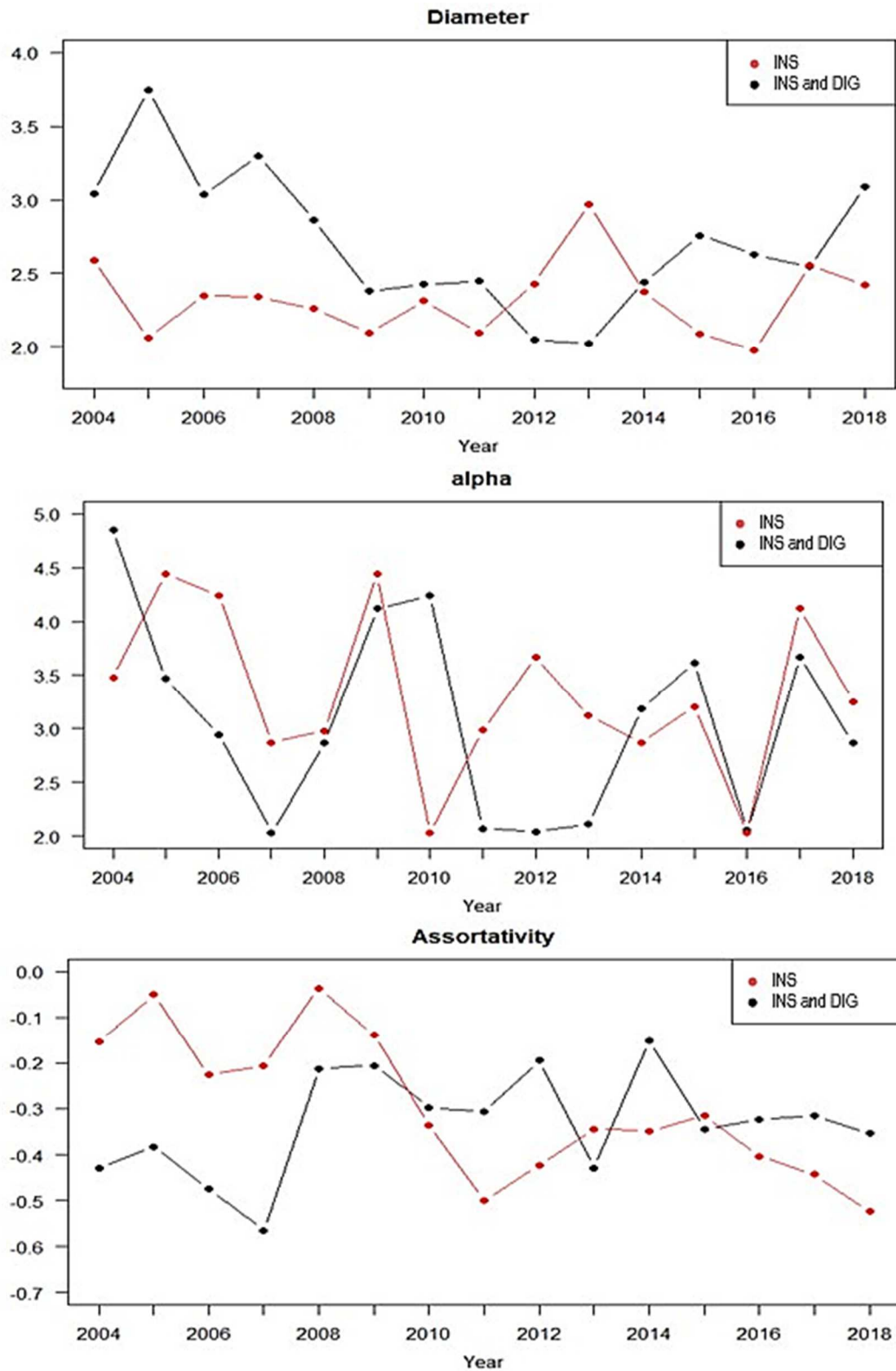


Figure 5. MST indicators for MST base multidimensional distance DTW from the period in four-year windows
 Source: own elaboration in RStudio.

CONCLUSIONS

Research in the insurance sector must consider digitalisation indicators that reflect both technological advancements and consumer behaviour. Due to the delay with which the data are issued our empirical

analysis covered the years 2004-2018. This research demonstrates the impact of digitalisation on the similarity of insurance sectors across EU countries during this period.

The applied methodology, constructing minimum spanning trees (MSTs) in three distinct ways, enables the assessment of dependency structures on an annual basis, over four-year periods, and cumulatively across the entire 2004-2018 timeframe.

MST topological indicators allow for the identification of crisis periods and facilitate comparisons of network connection structures during these critical moments.

Time series analysis of MST topological indicators, incorporating digitalisation data via the first method (based on the Mahalanobis distance), provides an annual overview of MST structures. These MSTs, reflecting the similarity of insurance sectors across seventeen dimensions, reveal structures that are conducive to systemic risk propagation prior to crises. In contrast, the second method, based on multivariate dynamic time warping (DTW) distance, shows relatively stable similarities over longer periods, with distinct behaviours emerging during crisis years. The resulting MST structures suggest a slower potential pace of financial shock propagation within the sector. The third method, which directly analyses the dynamics of topological indicators, effectively captures structural shifts before and during crises, with MST configurations varying depending on the period in question.

MST topological indicators exhibit strong sensitivity to structural dynamics, showing characteristic patterns both before and during financial crises. Thus, they hold promise as predictive tools in the analysis of global financial disruptions.

In this study, we defined 'similar countries' as those whose populations exhibit comparable behaviours regarding insurance activity and everyday life, influenced by modern technologies, digitalisation, and Internet use. We assessed this similarity in three different ways, each reflecting a distinct approach to measuring it. When comparing multivariate time series, similarity is typically calculated by transforming several variables into vectors in a multidimensional space and computing the Euclidean distance between them. The DTW distance measure allows for nonlinear alignment of one series to another, minimising the temporal distance between them.

Since the early twenty-first century, DTW has been widely used in data analysis tasks, including classification, clustering, and anomaly detection in time series data. Our research opens up new avenues for further study, including the potential application of this methodology for grouping EU countries based on the developmental dynamics of their insurance sectors, as well as exploring the methodological aspects of using multivariate DTW distances to assess the similarity of MST topological indicator time series constructed using various techniques.

The conclusions drawn from this study highlight the critical role of digitalisation in shaping the systemic risk profile of the EU insurance sector. The methodology applied, which incorporates minimum spanning trees (MSTs) constructed using the Mahalanobis distance and dynamic time warping (DTW), reveals significant structural patterns that inform about both the assessment of risk contagion pathways and the long-term stability of insurance networks. These insights not only enhance our understanding of systemic risk transmission but also provide valuable guidance for policymakers and industry stakeholders aiming to strengthen the resilience of the insurance sector.

There are several implications for the Insurance Sector. Firstly, the impact of digitalisation: the study demonstrates that digitalisation significantly influences the systemic risk profile of the EU insurance sector, particularly during periods of financial crises. High levels of digital integration can increase the similarity among insurance markets, potentially amplifying the transmission of financial shocks across borders. Next, we found international dependencies: the use of MST and DTW models reveals that insurance sectors in countries with similar digitalisation levels are more prone to shared systemic risks, highlighting the need for coordinated regulatory oversight at the international level. Then, we discovered risk transmission channels: the structure of connections within EU insurance sectors, as represented by MST models, suggests that high degrees of similarity can act as indirect channels for systemic risk propagation, increasing the speed and extent of financial contagion. Finally, we exposed crisis sensitivity: the analysis of MST topological indicators shows that sectors with more centralised network structures are more vulnerable to rapid changes in market conditions, potentially accelerating the spread of systemic risk during financial disruptions. On the other hand, we can also enumerate implica-

tions for insurance policies. On the one hand, in risk management, the high degree of interconnectedness observed in digitally advanced insurance sectors emphasises the need for more sophisticated risk management strategies that account for both local and cross-border risk exposures. On the other hand, for product personalisation digitalisation enables more precise tailoring of insurance products to individual customer needs, but also requires a greater focus on systemic risk management as global interconnections grow. As far as innovation and risk are concerned, the introduction of new technologies, such as InsurTech and IoT-based products, may reduce operational costs but also increase systemic vulnerabilities, necessitating a balanced approach to innovation. Finally, a conclusion for portfolio diversification: insurers should consider diversifying their product portfolios to mitigate the impact of systemic crises that can spread more rapidly in a highly interconnected, digital insurance environment.

These findings reinforce the importance of integrating digitalisation indicators into systemic risk assessments and underline the need for a more resilient and diversified insurance sector capable of withstanding future global financial shocks.

REFERENCES

- Acharya, V., Pedersen, L., Philippon, T., & Richardson, M.P. (2017). Measuring systemic risk. *The Review of Financial Studies*, 30(1), 2-47. <https://doi.org/10.1093/rfs/hhw088>
- Adrian, T., & Brunnermeier, M.K. (2016). CoVaR. *American Economic Review*, 106(7), 1705-1741. <https://doi.org/10.1257/aer.20120555>
- Albrecher, H., Bommier, A., Filipović, D., Koch-Medina, P., Loisel, S., & Schmeiser, H. (2019). Insurance: Models, digitalization, and data science. *European Actuarial Journal*, 9, 349-360. <https://doi.org/10.1007/s13385-019-00211-6>
- Albrecher, H., Bauer, D., Embrechts, P., Filipović, D., Koch-Medina, P., Korn, R., Loisel, S., Pelsser, A., Schiller, F., Schmeiser, H., & Wagner, J. (2018). Asset-liability management for long-term insurance business. *European Actuarial Journal*, 8, 9-25. <https://doi.org/10.1007/s13385-017-0165-2>
- Albrecher, H., Embrechts, P., Filipović, D., Harrison, G., Koch, P., Loisel, S., Vanini, P., & Wagner, J. (2016). Old-age provision: Past, present and future. *European Actuarial Journal*, 6, 287-306. <https://doi.org/10.1007/s13385-016-0133-3>
- Albert, R., & Barabási, A.-L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics*, 74, 47-97. <https://doi.org/10.1103/RevModPhys.74.47>
- Andrzejak, J., Chmielewski, L. J., Landmesser-Rusek, J., & Orłowski, A. (2024). The impact of the measure used to calculate the distance between exchange rate time series on the topological structure of the currency network. *Entropy*, 26(4), 279. <https://doi.org/10.3390/e26040279MDPI>
- Bellman, R., & Kalaba, R. (1959). On adaptive control processes. *IRE Transactions on Automatic Control*, 4(2), 1-9. <https://doi.org/10.1109/TAC.1959.1104822>
- Bekiros, S., Nguyen, D.K., Junior, L., & Uddin, G.S. (2017). Information diffusion, cluster formation and entropy-based network dynamics in equity and commodity markets. *European Journal of Operational Research*, 256(3), 945-961. <https://doi.org/10.1016/j.ejor.2016.07.032>
- Billio, M., Getmansky, M., Lo, A.W., & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3), 535-559. <https://doi.org/10.1016/j.jfineco.2011.12.010>
- Bisias, D., Flood, M., Lo, A.W., & Valavanis, S. (2012). A survey of systemic risk analytics. *Annual Review of Financial Economics*, 4, 255-296. <https://doi.org/10.1146/annurev-financial-110311-101754AnnualReviews>
- Boccara, N. (2003). *Modeling complex systems*. Springer. <https://doi.org/10.1007/b97378SpringerLink>
- Brownlees, C.T., & Engle, R.F. (2017). SRISK: A conditional capital shortfall measure of systemic risk. *The Review of Financial Studies*, 30(1), 48-79. <https://doi.org/10.1093/rfs/hhw060>
- Cedras, C., & Shah, M. (1995). Motion-based recognition: A survey. *Image and Vision Computing*, 13(2), 129-155. [https://doi.org/10.1016/0262-8856\(95\)99722-5](https://doi.org/10.1016/0262-8856(95)99722-5)
- Chan-Lau, J. (2010). Regulatory capital charges for too-connected-to-fail institutions: A practical proposal. *IMF Working Paper 10/98*. International Monetary Fund. Retrieved from <https://www.imf.org/external/pubs/ft/wp/2010/wp1098.pdf> on May 2, 2024.

- Denkowska, A., & Wanat, S. (2021). A Dynamic MST-delta CoVaR Model of Systemic Risk in the European Insurance Sector. *Statistics in Transition: New Series*, 22(2), 173-188. <https://doi.org/10.21307/stattrans-2021-017>
- Denkowska, A., & Wanat, S. (2020a). Dependencies and systemic risk in the European insurance sector: New evidence based on Copula-DCC-GARCH, model and selected clustering methods. *Entrepreneurial Business and Economics Review*, 8(4), 7-27. <https://doi.org/10.15678/EBER.2020.080401>
- Denkowska, A., & Wanat, S. (2020b). A Tail Dependence-Based MST and Their Topological Indicators in Modeling Systemic Risk in the European Insurance Sector. *Risks*, 8(2), 39, 1-22. <https://doi.org/10.3390/risks8020039>
- Denkowska, A., & Wanat, S. (2021). Dynamic Time Warping Algorithm in Modeling Systemic Risk in European Insurance Sector. *Entropy*, 23(8), 1-18. <https://doi.org/10.3390/e23081039>
- Denkowska, A., & Wanat, S. (2022). Linkages and systemic risk in the European insurance sector: Some new evidence based on dynamic spanning trees. *Risk Management*, 24(2), 123-136. <https://doi.org/10.1080/13669877.2021.1956014>
- Denkowska, A., Wanat, S., & Vieito, J.P. (2023). The Effect of the 2004 EU Enlargement on the Development and Similarity of the Insurance Sectors in the EU. *European Review*, 2023, 1-24. <https://doi.org/10.1017/S1062798723000131>
- Diebold, F.X., & Yilmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1), 119-134. <https://doi.org/10.1016/j.jeconom.2014.04.011>
- EIOPA. (2017). Investment behavior report. EIOPA-BoS-17/230, 16 November 2017. Retrieved from <https://www.eiopa.europa.eu/> on November 16, 2023.
- EIOPA (2024). EIOPA's Report on the Digitalisation of the European Insurance Sector. Retrieved from <https://www.eiopa.europa.eu/> on November 16, 2023.
- Eling, M., & Lehmann, M. (2018). The Impact of Digitalization on the Insurance Value Chain and the Insurability of Risks. *The Geneva Papers on Risk and Insurance – Issues and Practice*, 43, 359-396. <https://doi.org/10.1057/s41288-018-0097-2>
- Franses, P.H., & Wiemann, T. (2020). Intertemporal similarity of economic time series: An application of Dynamic Time Warping. *Computational Economics*, 56(1), 59-75. <https://doi.org/10.1007/s10614-019-09839-x>
- Geiger, D., Gupta, A., Costa, L.A., & Vlontzos, J. (1995). Dynamic programming for detecting, tracking, and matching deformable contours. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 17(3), 294-302. <https://doi.org/10.1109/34.391786>
- Gray, D., & Jobst, A. (2013). Systemic Contingent Claim Analysis – Estimating Market-Implied Systemic Risk. *IMF Working Paper, No. 13/54*. Retrieved from <https://www.imf.org/en/Publications/WP/Issues/2020/03/01/Systemic-Contingent-Claim-Analysis-Estimation-Market-Implied-Systemic-Risk-406651> on May 2, 2024.
- Harrington, S.E. (2009). The financial crisis, systemic risk, and the future of insurance regulation. *Journal of Risk and Insurance*, 76(4), 785-819. <https://doi.org/10.1111/j.1539-6975.2009.01314.x>
- Hilbert, M. (2020). Digital technology and social change: the digital transformation of society from a historical perspective. *Dialogues in Clinical Neuroscience*, 22(2), 189. <https://doi.org/10.31887/DCNS.2020.22.2/mhilbert>
- IMF. (2016). The insurance sector – Trends and Systemic Risk implications. *IMF – Global Financial Stability Report 2016*, Chapter 3. Retrieved from <https://www.imf.org/en/Publications/GFSR/Issues/2016/05/06/Global-Financial-Stability-Report-May-2016> on May 2, 2024.
- IMF. (2020). Cyber Risk and Financial Stability. *IMF – Global Financial Stability Report 2020*. Retrieved from <https://www.imf.org/en/Publications/GFSR/Issues/2020/10/15/global-financial-stability-report-october-2020> on May 2, 2024.
- Jajuga, K., Karaś M., Kuziak, K., & Szczepaniak, W. (2017). Ryzyko systemu finansowego. Metody oceny i ich weryfikacja w wybranych krajach. *Materiały i Studia*, 329, Narodowy Bank Polski. Retrieved from <https://www.nbp.pl> on May 2, 2024.
- Jajuga, K. (2023). Data Analysis for Risk Management – Economics, Finance and Business: New Developments and Challenges. *Risks*, 11(4), 70. <https://doi.org/10.3390/risks11040070>
- Jobst, A.A. (2014). Systemic risk in the insurance sector: a review of current assessment approaches. *Geneva Papers on Risk and Insurance*, 39(3), 440-470. <https://doi.org/10.1057/gpp.2014.5>
- Kaur, P., & Singh, M. (2023). Exploring the impact of InsurTech adoption in Indian life insurance industry: a customer satisfaction perspective. *The TQM Journal*, forthcoming. <https://doi.org/10.1108/TQM-06-2023-0301>

- Keogh, E.J., & Pazzani, M.J. (2000). Scaling up dynamic time warping for data mining applications. *Proceedings of the sixth ACM SIGKDD international conference on Knowledge discovery and data mining*, 285-289. <https://doi.org/10.1145/347090.347161>
- Kruskal, J. (1956). On the shortest spanning subtree of a graph and the traveling salesman problem. *Proceedings of the American Mathematical Society*, 7(1), 48-50. <https://doi.org/10.1090/S0002-9939-1956-0079333-3>
- Kwon, W.J., & Wolfrom, L. (2017). Analytical tools for the insurance market and macro-prudential surveillance. *OECD Journal: Financial Market Trends*, 2016(1), 1-47. <https://doi.org/10.1787/fmt-2016-5jrs7d94kv1d>
- Li, W., Hommel, U., & Paterlini, S. (2018). Network topology and Systemic Risk: Evidence from the Euro Stoxx market. *Finance Research Letters*, 27, 105-112. <https://doi.org/10.1016/j.frl.2018.03.007>
- Lyons, D. (2019). Insurance 4.0: Six Winning Strategies in the Fourth Industrial Revolution, Digital Insurance, RGA Retrieved from <https://www.rgare.com/knowledge-center/article/insurance-4.0-six-winning-strategies-in-the-fourth-industrial-revolution> on May 2, 2024.
- Masiello, M.P. (2020). Insurance Agency 4.0: Prepare Your Agency for the Future; Develop Your Road Map for Digitalization; Increase Profit, Scalability and Time. Autor Academy Elite, ISBN 1647465222, 9781647465223
- Mantegna, R.N. (1999). Hierarchical structure in financial markets. *European Physical Journal B – Condensed Matter and Complex Systems*, 11(1), 193-197. <https://doi.org/10.1140/epjb/e20010>
- Nicoletti, B. (2020). Insurance 4.0: Benefits and Challenges of Digital Transformation. Palgrave Macmillan Cham Insurance 4.0: Benefits and Challenges of Digital Transformation, Palgrave Macmillan (2020). <https://doi.org/10.1007/978-3-030-48874-1>
- Newman, M. (2002). Assortative mixing in networks. *Physical Review Letters*, 89, 11. <https://doi.org/10.1103/PhysRevLett.89.218701>
- OECD (2017). Enhancing the Role of Insurance in Cyber Risk Management, OECD Publishing, Paris. Enhancing the Role of Insurance in Cyber Risk Management, OECD Publishing, Paris (2017). <https://doi.org/10.1787/9789264282148-en>
- Petitjean, F., Ketterlin, A., & Gançarski, P. (2011). A global averaging method for dynamic time warping, with applications to clustering. *Pattern Recognition*, 44(3), 678-693. <https://doi.org/10.1016/j.patcog.2010.09.010>
- Rak, R., & Rak, E. (2020). The Fractional Preferential Attachment Scale-Free Network Model. *Entropy*, 22, 509. <https://doi.org/10.3390/e22050509>
- Ronken, L. (2018). Industry 4.0 – Implications for the Insurance Industry, Insurance Issues, General Reinsurance AG, Cologne. Industry 4.0 – Implications for the Insurance Industry, Insurance Issues, General Reinsurance AG
- Sakoe, H., & Chiba, S. (1978). Dynamic programming algorithm optimization for spoken word recognition. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 26(1), 43-49. <https://doi.org/10.1109/TASSP.1978.1163057>
- Shokoohi-Yekta, M., Hu, B., Jin, H., Wang, J., & Keogh, E.J. (2017). Generalizing Dynamic Time Warping to the Multidimensional Case Requires an Adaptive Approach. *Data Mining and Knowledge Discovery*, 31(1), 1-31. <https://doi.org/10.1007/s10618-016-0483-7>
- Schwab, K., & Davis, N. (2018). *Shaping the Future of the Fourth Industrial Revolution*. World Economic Forum
- Torri, G., Giacometti, R., & Paterlini, S. (2018). Robust and Sparse Banking Network Estimation. *European Journal of Operational Research*, 270(1), 51-65. <https://doi.org/10.1016/j.ejor.2017.11.031>
- Waliszewski, K., Cichowicz, E., Gębski, Łukasz, Kliber, F., Kubiczek, J., Niedziółka, P., Solarz, M., & Warchlewska, A. (2024). Digital loans and buy now pay later from LendTech versus bank loans in the era of ‘black swans’: Complementarity in the area of consumer financing. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 19(1), 241-278. <https://doi.org/10.24136/eq.2982>
- Wang, G.-J., Xie, C., Han, F., & Sun, B. (2012). Similarity measure and topology evolution of foreign exchange markets using dynamic time warping method: Evidence from minimal spanning tree. *Physica A: Statistical Mechanics and Its Applications*, 391(16), 4136-4146. <https://doi.org/10.1016/j.physa.2012.02.014>
- Wang, G.J., Xie, Ch., Zhang, P., Han, F., & Chen, S. (2014). Dynamics of Foreign Exchange Networks: A Time-Varying Copula Approach. *Hindawi Publishing Corporation Discrete Dynamics in Nature and Society*, 2014, Article ID 170921. <https://doi.org/10.1155/2014/170921>
- Zhao, J., Xi, Z., Itti, L. (2016). MetricDTW: local distance metric learning in Dynamic Time Warping. Retrieved from <https://arxiv.org/abs/1606.03628> on May 2, 2024.


Authors

The contribution share of authors is equal and amounts to $\frac{1}{3}$ for each of them.

Anna Denkowska (corresponding author)

Assistant Professor in the Mathematics Department of the Krakow University of Economics (Poland). MSc in mathematics (Jagiellonian University), PhD in mathematics (Jagiellonian University). Her research interests include functional analysis, algebraic topology, PDE's, statistics, risk modelling, and network theory.

Correspondence to: Dr Anna Denkowska, Krakow University of Economics, Department of Mathematics, Rakowicka 27, 31-510 Krakow, Poland, e-mail: anna.denkowska@uek.krakow.pl

ORCID  <https://orcid.org/0000-0003-4308-8180>

Stanisław Wanat

Associate Professor in the Mathematics Department at the Krakow University of Economics (Poland), MSc in mathematics (Jagiellonian University), PhD in economics (Krakow University of Economics), habilitation in economics (Krakow University of Economics). His research interests include statistics, actuarial methods, risk modelling, and forecasting.

Correspondence to: Prof. UEK, dr hab. Stanisław Wanat, Krakow University of Economics, Department of Mathematics, Rakowicka 27, 31-510 Krakow, Poland, e-mail: wanats@uek.krakow.pl

ORCID  <https://orcid.org/0000-0001-6225-4207>

João Paulo Vieito

Associate Professor at the Polytechnic Institute of Viana do Castelo (Portugal), MSc in Finance (Catholic University of Porto), PhD in Management Sciences, Finance specialisation, Faculty of Economics (University of Porto), Managing Director at the School of Business Sciences at Polytechnic Institute of Viana do Castelo, Main chairman of the World Finance Conference. His research interests include business administration, gender, gender studies, gender equality, gender differences, neuroeconomics, neurofinance, behavioural finance, gender and development, capital Structure, stocks, sociology, markets, economy, finance, mergers, investment, banking, and financial risk management.

Correspondence to: Prof. João Paulo Vieito, School of Business Studies, Polytechnic Institute of Viana do Castelo, Av. Pinto da Mota, 4930-600 Valença, Portugal, e-mail: joaovieito@esce.ipv.pt

ORCID  <https://orcid.org/0000-0002-0899-4930>

Acknowledgements and Financial Disclosure

Authors have received research support from the National Science Centre (NCN), Poland, grant No 2024/53/B/HS4/03258.

Use of Artificial Intelligence

This text is free of AI/GAI usage.

Conflict of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Copyright and License



This article is published under the terms of the Creative Commons Attribution (CC BY 4.0) License <http://creativecommons.org/licenses/by/4.0/>

Published by Krakow University of Economics – Krakow, Poland

